VOLUME XLVIII NUMBER 3

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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Solar Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
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HENRY G. GALE
Major, U.S.A. in France

OCTOBER 1918

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London and Edinburgh
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

October 1916 Vol. XLVIII, No. 3

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For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, 23*s.* each; single copies, including postage, 3*s.* 3*d.* each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 11.25 each; single copies, including postage, Yen 1.30 each.

For China: The Mission Book Company, 18 Peking Road, Shanghai. Yearly subscriptions, $5.00; single copies, 65 cents, or their equivalents in Chinese money. Postage extra, if mailed direct outside of Shanghai, on yearly subscriptions 62 cents, on single copies 11 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit. Deliveries are not guaranteed in the war zones of Europe, nor in other countries where transportation is disturbed by conditions incident to war.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of The Astrophysical Journal, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address of the Editorial Office is "Arcturus, Chicago."

The articles in this Journal are indexed in the *Readers' Guide Supplement*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of March 3, 1879.

Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

THE
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VOLUME XLVIII OCTOBER 1918 NUMBER 3

ON THE EXCESS OF OUTWARD MOTION OF THE STARS OF CLASS B

By C. D. PERRINE

The residual velocities of the B stars, after eliminating the solar motion, show a strong positive tendency, so much so that it gave rise to the belief that there was some sort of a constant error in the positions of the lines of their spectra, or a pressure-effect in the atmospheres of these stars which caused their velocities to be observed too large on the average by +4 or +5 km.

The preference of the B stars for the Galaxy and their distance, together with the hypothesis that their great brightness and peculiar spectral condition result from the action of cosmical matter in those regions, suggest an explanation for this excess of outward motion. It is that a process of selection has been at work, that the B stars are those which have penetrated the region richest in such cosmical matter, and that the observer being on the inside of such a ring of matter these stars, or at least the nearer ones, would have a preference for outward motion. Such a theory requires that only the stars which were moving outward would have passed into this ring of matter and become B-type stars, if we assume that the ring of matter is very extensive. If not of so great extent, then some stars could have entered from the outside. Such stars

would in general be approaching the observer and would be fainter. If such is the true condition, then the brighter and nearer stars of class B should show almost entirely outward motion, and the fainter and more distant of these stars might contain a considerable proportion of approaching stars. This condition would be modified somewhat if the matter were not circular but spiral, for example. In a previous investigation[1] attention was drawn to the fact that there was a progressive change in the residual velocities of stars of class B, the brighter ones showing an excess of outward motion, whereas the fainter stars showed an excess of inward motion. These results were from velocities in which the constant error term K had been removed. As later evidence has been found[2] tending to show that this term K is really an excess of outward motion the foregoing progression indicates not inward motion for the fainter stars but a *smaller* excess of outward motion for these stars. A re-examination was made of these B stars in which they were classified with respect to size of proper motion also. The results are given in Table I.

On account of the well-known differences of the later B stars only classes B to B5 inclusive were used. Three facts are noticeable in these results:

a) Only one star brighter than magnitude 3.0 is approaching, and that is barely over the line (−1 km).

b) The proportion of approaching stars increases toward the fainter magnitudes and is greater for the stars of smaller μ.

c) The residual velocities in the stars of smaller μ decrease toward the fainter stars. This decrease is due to the increasing proportion of stars of approach among the fainter stars and not to a general decrease of outward motion, as is seen from the receding stars which are given just underneath the results for both classes in Table I.

Adams' list of the radial velocities of 500 stars[3] contains 27 stars of classes B to B5 inclusive. With two exceptions they are all fainter than magnitude 5.0, having a mean of 5.4. Of these 10, or 37 per cent, are approaching. The residual velocity of the 27

[1] *Astrophysical Journal*, **41**, 319, 1915. [2] *Ibid.*, **44**, 244, 1916.

[3] *Mt. Wilson Contr.*, No. 105; *Astrophysical Journal*, **42**, 172, 1915.

TABLE I

Residual Radial Velocities of Stars of Classes B–B5*

	μ = 0 to 0".029						μ = 0".030 and Over					
	Mean Mag.	Residual Velocity V	μ	No. of Stars	Approaching Stars		Mean Mag.	Residual Velocity V	μ	No. of Stars	Approaching Stars	
					No.	Percentage					No.	Percentage
		km						km				
2.2 and brighter....	1.9	+ 6.4	0".007	6	0	0	1.6	+ 5.5	0".060	9	1	.....
2.3 to 2.9.....	2.7	+11.5	.014	5	0	0	2.8	+ 7.6	.045	7	0	0
3.0 to 3.9.....	3.6	+ 7.4	.016	22	1	5	3.5	+ 5.2	.046	16	3	19
		+ 8.3		21				+ 7.4		13		
4.0 to 4.9.....	4.6	+ 3.9	.013	59	17	29	4.5	+ 6.0	.043	31	7	23
		+ 8.7		42				+ 8.9		24		
5.0 and fainter.....	5.2	+ 1.7	.017	16	7	44	5.3	+ 5.9	.040	8	2	25
		+ 9.3		9				+10.0		6		

* These residual velocities resulted from the use of a solar velocity of 19.5 km toward $\alpha = 270°, \delta = +30°$.

is +2.2 km and of the 17 receding stars +9.3 km. These agree well with the results in Table I.

The results in Table I are shown graphically in Fig. 1, where the curves *A* and *C* relate to magnitude, *B* and *D* to μ.

The foregoing results led to an examination of the galactic stars of other spectral classes with a view to discovering if this preference of approaching motions for the fainter and more distant stars is a general phenomenon or is confined to the B stars. Like

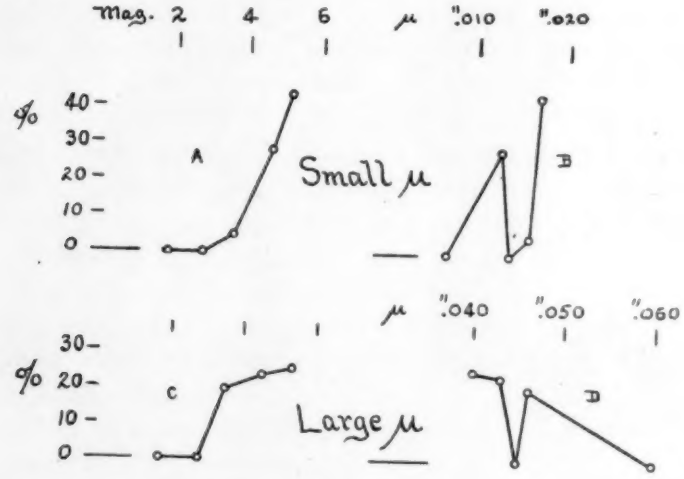


FIG. 1

the preceding this examination was first limited to Campbell's L.O. catalogues because they represent all parts of the sky. Only the *numbers* of stars are considered in this case, the residual velocities being left for later investigation, largely on account of the few stars available in some of the spectral classes. The results are given in Table II. They show that there is essentially no preference of the galactic stars of spectral classes A, F, G, K, and M for positive velocities, the distributions being as uniform between positive and negative velocities as can be expected from the nature of the data. The number of the brighter stars in class M is too small for the predominance of positive velocities to have any significance in the face of the uniformity shown by the larger number of

the fainter stars of the same class. The intermediate position of the B8 and B9 stars is noticeable but of small weight on account of their small number. I conclude, therefore, that among the 852 galactic stars examined only the B-type stars show a numerical preference for receding velocities, all of the other types showing no preference either for receding or approaching motions.

TABLE II

ACCORDING TO DIRECTION OF MOTION

CAMPBELL'S L.O. CATALOGUES. GALACTIC

SPECTRAL CLASS	3.9 AND BRIGHTER				4.0 AND FAINTER				TOTAL NO. OF STARS
	Receding		Approaching		Receding		Approaching		
	No.	Percentage	No.	Percentage	No.	Percentage	No.	Percentage	
B–B5.....	60	92	5	8	81	71	33	29	179
B8–B9.....	5	83	1	17	16	59	11	41	33
A.....	10	36	18	64	41	49	42	51	111
F.....	13	52	12	48	43	47	48	53	116
G.....	5	33	10	67	36	49	38	51	89
K.....	42	49	43	51	103	52	94	48	282
M.....	(9	82)	(2	18)	17	55	14	45	42

Among the stars of class A of magnitude 3.9 and brighter there is a preponderance of negative velocities, but the number of stars is too small to justify further consideration at the present time.

An examination of the stars in the non-galactic regions appears to show essentially the same distributions with respect to the numbers of stars receding and approaching as in the galactic regions for the classes later than B. In consequence of this indication the 500 stars of Adams' Mount Wilson catalogue were classified with respect to the numbers of stars having velocities of recession and approach, with the results given in Table III. The early B-type stars show a small preference for receding velocities, agreeing in general with the results from Campbell's catalogues. The A-type stars show a small preference for receding velocities but hardly enough to justify remark. The later B and the classes F, G, K, and M may safely be concluded to show no real preference.

The results of Tables II and III for the approaching stars are shown graphically in Fig. 2. Curves *A* and *B* refer to the stars of magnitude 3.9 and brighter and 4.0 and fainter, respectively, of Campbell's L.O. catalogues. Curve *C* refers to the stars of Adams' Mount Wilson catalogue. The curves for the receding velocities

TABLE III

ACCORDING TO DIRECTION OF MOTION
ADAMS' MOUNT WILSON CATALOGUE

SPECTRAL TYPES	RECEDING		APPROACHING		TOTAL NO. OF STARS
	No.	Percentage	No.	Percentage	
B–B5.....	17	63	10	37	27
B6–B9.....	34	53	30	47	64
A.....	80	58	58	42	138
F.....	17	45	21	55	38
G.....	47	49	49	51	96
K.....	39	45	48	55	87
M.....	23	51	22	49	45

are simply reversals of these curves. An examination of the velocities of approach shows a larger value for the more distant stars of class B–B5 than for the nearer ones. The results are given in Table IV.

TABLE IV

	SMALL μ				LARGE μ			
	Mag.	No. of Stars	$\bar{V}$	$\bar{\mu}$	Mag.	No. of Stars	$\bar{V}$	$\bar{\mu}$
Campbell's L.O. catalogues			km				km	
$3^{M}0$ to $3^{M}9$.....	3.8	1	−11.2	0″.010	3.4	3	−4.2	0″.051
4.0 to 4.9.....	4.5	17	− 7.8	.015	4.4	7	−4.1	.047
5.0 and fainter....	5.2	7	− 8.1	.017	5.6	2	−6.4	.040
Adams' Mount Wilson catalogue...	5.6	6	−11.3	.020	4.5	4	−7.7	.042
Mean.....	.....	31	− 8.7	.....	.....	16	−5.3	.....

This would be in accordance with the theory that the approaching stars of early class B have entered the ring of cosmical matter from the outside and that their velocities have been retarded. The

amount of data is too small, however, to place any great confidence in the result, notwithstanding its apparent consistency, or to discuss its bearing further at the present time.

The results in Table I show a progressive increase in the *proportion* of stars with negative velocities, as the stars become fainter. Whether this is real or whether it is really an effect of distance only

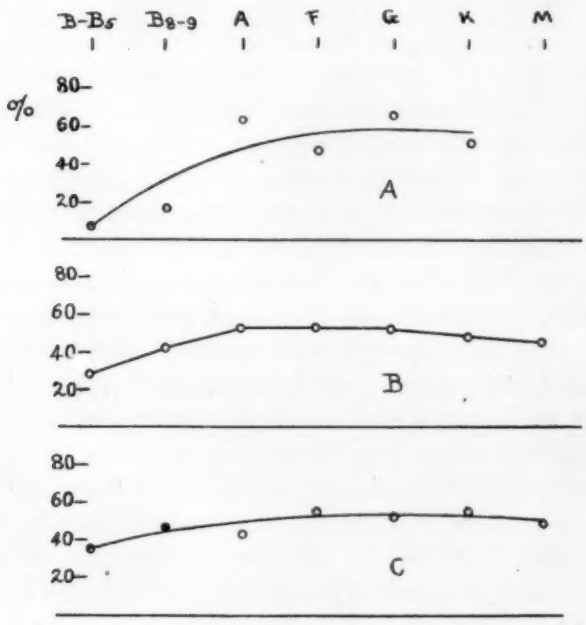


FIG. 2

cannot be determined at present. The proper motions of the stars of class B are all so small that there must be great uncertainty in them and it may be that magnitude is, in this case, a better gauge of distance than the size of proper motion. Direct determinations by parallactic displacement for these stars seem hopelessly beyond our present means. In Table I the positive velocities considered alone show a small increase with decreasing magnitude. Table IV indicates a similar effect in the negative velocities considered separately. The amount of material, however, seems to be too small to justify classification with respect to ellipsoidal regions

which were shown in a recent paper[1] to appear to have caused some of the magnitude-velocity effect.

The residual velocities used in this paper may be considered to be derived using a solar motion of 20 km toward $\alpha = 270°$, $\delta = +30°$. The directions and velocities actually used differ slightly in some cases from the above but are generally within ½ km and 1° of these.

BEARING OF THIS INVESTIGATION ON K TERM

The results of this investigation have a bearing upon the interpretation to be put upon the term K which has been found in the solutions for solar motion from radial velocities. This term may be defined as the excess of velocity with respect to algebraic sign after the elimination of solar motion. Campbell found large positive values for this term for the spectral classes B, K, and M and no appreciable values for the stars of classes A, F, and G, from his solutions from about 1200 stars in all parts of the sky.[2] Using essentially the same apex of solar motion, Adams from his 500 stars found a much reduced value for B, negative (generally small) values for A, F, and K, and negligibly small positive values for classes G and M.[3]

The writer found[4] that a few regions (about one-third of the whole) gave consistently larger positive velocities in the three spectral classes B, K, and M, and that if these were omitted there remained little or no positive residuals for the remaining two-thirds of the stars.

The results obtained in the first part of the present paper indicate a fundamental difference in the distributions of the velocities of recession and approach between the stars of class B and the later spectral classes. Interpreted directly, this further indicates in my opinion a lack of sufficient constancy for the term K to be ascribed to any physical cause and to strengthen the conclusion that these excess velocities are chiefly motion.

[1] *Astrophysical Journal*, **46**, 266, 1917.

[2] *Lick Observatory Bulletin*, **6**, 104, 127, 1911.

[3] *Mt. Wilson Contr.*, No. 105, p. 20.

[4] *Astrophysical Journal*, **44**, 244, 1916.

CONCLUSIONS

I. The stars of class B, particularly those early in the class, after the elimination of solar motion, show a marked preponderance of outward motions.

II. The motions of the B stars of magnitude 2.9 and brighter which were investigated are essentially all *outward*. In the stars of magnitude 3.0 and fainter the proportion of stars with velocities of approach increase for the fainter magnitudes and with the smaller proper motions.

III. The average velocities of recession appear to be essentially the same for all distances, as indicated by size of proper motion, but to increase slightly with decrease of brightness.

IV. In the spectral classes A, F, G, K, and M no preference is shown for either velocities of recession or approach in either galactic or non-galactic regions.

V. The observed distributions of the velocities of approach and recession in the B stars can be explained upon the hypothesis that the peculiar spectral condition of the B-type stars is due chiefly to a ring of distant galactic cosmical matter, and that the nearer and brighter of these stars have entered this ring in general from the inside, whereas a considerable proportion of the fainter and more distant ones have entered from the outside.

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA
January 29, 1918

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

SEVENTH PAPER: THE DISTANCES, DISTRIBUTION IN SPACE, AND DIMENSIONS OF 69 GLOBULAR CLUSTERS

By HARLOW SHAPLEY

I. PARALLAXES FROM VARIABLE STARS, APPARENT MAGNITUDES, AND ANGULAR DIAMETERS

Applying the methods discussed in the preceding *Contribution*,[2] the parallaxes of a few clusters are obtained directly from the periods and magnitudes of Cepheid variables; the parallaxes of a considerably larger number are derived from the mean magnitudes of the brightest cluster stars, and the survey is then made complete through measures of diameters of the photographic images of all globular systems. In Table I are listed the clusters for which the variable stars have received a sufficiently detailed discussion to permit a determination of the parallax directly from the luminosity-period curve of Cepheid variation. The apparent diameters and the adopted parallaxes are taken from tables appearing on following pages; the method of weighting the results is also subsequently described.

The parallaxes for Messier 3, 5, 15, and 22, depending almost entirely on the median magnitudes of numerous variable stars, are the most accurate. The computed probable error of the absolute magnitude is ± 0.2; that of the apparent magnitude is estimated to vary from ± 0.05 for Messier 3 to possibly ± 0.2 for Messier 22. The corresponding limits of the probable error of the parallaxes are $\pm 0''.000007$ and $\pm 0''.000015$, that is, 10 and 13 per cent, respectively.

For most other clusters, of course, the errors are somewhat greater, particularly for those where the parallax depends solely upon either the magnitudes of the brightest stars or the measured

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 152.

[2] *Mt. Wilson Contr.*, No. 151.

diameter on photographic charts. For the former the estimated average probable error is 20 per cent, for the latter 25 per cent.[1] When two or three sources are available, as in Tables I and V, the errors are less; but in Table I the different parallaxes for each cluster are not all completely independent, as they were used in part to determine the reduction constants and curves. The residuals in the last column of Table I are expressed in millionths of a second of arc, and their minuteness indicates the validity of the methods involving diameters and the magnitudes of the bright stars.

TABLE I

COMPARISON OF CLUSTER PARALLAXES FROM VARIABLES, MAGNITUDES, AND DIAMETERS

DESIGNATION		APPARENT DIAMETER	PARALLAX (UNIT IS 0".000001)				RESIDUALS
N.G.C.	Messier		Adopted	From Variables	From Magnitudes	From Diameters	
5272	3	7'.0	72	72	71	72	0, − 1, 0
5904	5	8.6	80	80	80	81	0, 0, +1
6205	13	10.6	90	82:	89	91	−8:, − 1, +1
6656	22	16.0	118	116	121	116	−2, + 3, −2
7078	15	5.0	68	67	69	59	−1, + 1, −9
7089	2	7.0	64	65	60	72	+1, − 4, +8
5139	.....	30	153	150	170:	155	−3, +17:, +2
Small Magellanic Cloud	.....	.....	52	52	.....	.....	.....

The parallax of the Small Magellanic Cloud, which is given at the end of Table I, is relatively uncertain, for the value from variables can be checked by neither diameter measures nor maximum luminosities, and the zero-point error in the provisional magnitude scale used by Miss Leavitt is unknown.[2]

[1] These estimates appear to be safely conservative. After the smoothing operation, described on a later page, much smaller average errors are obtained for the final parallaxes in Tables V and VIII; thus, including the 10 per cent probable error in the parallaxes due to uncertainty of the zero-point of the absolute scale, the average probable error for all clusters is estimated to be less than 15 per cent. Cf. Table VII.

[2] See sec. III of the preceding *Contribution*. Previous values for the parallax of the Small Magellanic Cloud are: Hertzsprung, $\pi = 0''.0001$, *Astronomische Nachrichten*, **196**, 204, 1913; Kapteyn, $\pi = 0''.00004$, *Mt. Wilson Contr.*, No. 82, 71, 1914; Shapley, $\pi = 0''.00006$, *Mt. Wilson Contr.*, No. 116, 82, 1915. The new value in Table I is probably an improvement over the others because it allows for diversity in color of the variable stars and is based upon more definite knowledge of their absolute magnitudes.

In the preceding paper we found from the study of the bright stars and variables in several clusters that, after excluding the five brightest, the mean absolute photographic magnitude of the 25 most luminous objects is

$$M_{25} = -1.51 \pm 0.28.$$

Adopting this value, we have derived from measures of apparent magnitude the distances of practically all clusters north of declination $-30°$, the southern limit reached with the 60-inch reflector.

Of 300 cluster photographs taken during the last three years as a part of the program, about 175 have been measured for the magnitudes of either the bright stars or the variables; but the material is too extensive to describe in detail. Nearly all of the photographs were made with full aperture and on Seed 27 plates of various emulsions. The exposures vary in length from 10 seconds to 2 hours, but are mostly between 1 and 12 minutes in duration. For the clusters south of $-20°$ the altitude was frequently so low that plates with first-class images could not be secured. Mr. Hoge has assisted with all the observational work.

A summary of the work on each cluster is given in Table II. The designation in the first column is followed in the second with numbers indicating the total number of plates used in all phases of the work and those used in the derivation of the mean magnitude. For the latter purpose the plates, with few exceptions, involve direct polar comparisons on two or more nights; the images of between 50 and 100 cluster stars and of between 20 and 50 Polar Standards were measured at least twice on each plate; and the measures were corrected as usual for scale, distance from center, and differential atmospheric extinction. As the measures, reductions, and discussion cannot well be given for the individual clusters, the method of work is illustrated merely with a summary of the final magnitudes for the bright stars in Messier 2 (Table III) and Messier 53 (Table IV).

The radius of the concentric region in which all bright stars were measured, given in the third column of Table II, does not closely indicate the apparent size of the cluster either actually

TABLE II

Summary of Observations

N.G.C.	No. Plates	Photographic Magnitude 25 Stars				Weight	Angular Diameter			
		Radius	Mean	Av. Dev.	Extremes		Melotte	Davis	Shapley	Mean
288	2, 2	4′	14.81	±0.16	14.38–15.04	*d*	12′	4.′0	4.′4	4.′2
1904	3, 2	2	15.29	0.13	15.01–15.72	*d*	4.5	2.4	2.2	2.3
4147	4, 2	1.5	16.58	0.18	16.23–16.93	*d*	1	0.8	0.8	0.8
5024	2, 2	5	15.07	0.09	14.94–15.26	*b*	5	5.6	5.4	5.5
5139	3, 2	.....	(12.3:)	.....	.....	*d*	45	32	28	30
5272	65, 4	9	14.23	0.13	13.92–14.45	*a*	18	5.8	8.3	7.0
5904	3, 3	4	13.97	0.15	13.74–14.27	*a*	15	9.1	8.2	8.6
6093	2, 2	3	14.88	0.08	14.72–15.09	*d*	5	4.0	3.0	3.5
6121	3, 2	6	13.84	0.35	13.18–14.40	*c*	20	10.0	11.0	10.5
6205	15, 4	6	13.75	0.11	13.45–13.92	*b*	12	10.2	11.0	10.6
6218	2, 2	6	13.97	0.22	13.56–14.31	*b*	9	8.8	8.8	8.8
6229	3, 2	2.5	16.18	0.13	15.90–16.37	*d*	.....	1.0	1.2	1.1
6254	2, 2	5	14.06	0.24	13.35–14.38	*c*	10	10.9	10.0	10.4
6333	2, 2	4	15.61	0.16	15.08–15.88	*b*	3	3.0	3.5	3.2
6341	3, 3	7	13.86	0.13	13.60–14.16	*c*	8	8.4	7.6	8.0
6356	2, 2	1.5	17.16	0.14	16.86–17.44	*d*	1.5	2.0	1.8	1.9
6402	2, 2	5	15.44	0.24	14.85–15.86	*b*	6	3.1	3.8	3.4
6626	3, 3	3	14.87	0.16	14.49–15.11	*b*	5	5.2	4.2	4.7
6638	2, 2	2	16.22	0.19	15.90–16.60	*b*	1.5	1.6	1.8	1.7
6642*	3, 2	1	16.07	0.26	15.51–16.46	*c*	1	1.2	1.2	1.2
6656	6, 3	5.5	13.08	0.19	12.58–13.55	*a*	20	14.5	17.5	16.0
6712	5, 2	1.5	16.10	0.19	15.65–16.36	*d*	2.5	2.2	2.0	2.1
6779	6, 2	2	15.31	0.20	14.98–15.70	*c*	1.5	2.2	2.6	2.4
6864	2, 2	1	17.06	0.13	16.76–17.35	*c*	2	1.6	1.5	1.6
6934	5, 3	3	15.78	0.19	15.33–16.11	*c*	1.5	1.4	1.5	1.4
6981	7, 2	3	15.92	0.17	15.53–16.20	*b*	2	2.2	2.4	2.3
7078	7, 3	6	14.31	0.11	14.13–14.55	*a*	6	4.8	5.2	5.0
7089	7, 3	4	14.61	0.09	14.25–14.76	*b*	8	7.3	6.8	7.0
7099	4, 3	2.5	14.63	±0.′27	13.77–15.04	*c*	6	4.6	4.7	4.6

* See n. 1, p. 160.

or relatively. The basis of its choice has been described in the fifth section of the preceding paper.

TABLE III

MAGNITUDES OF BRIGHT STARS IN MESSIER 2 (N.G.C. 7089)

STAR*	PHOTOGRAPHIC MAGNITUDES				RESIDUALS†	DEVIATION
	3883P	3902P	3904P	Means		
1.....	14.09	14.16	14.04	Bright	.....	.....
2.....	14.68	14.59	14.53	14.60	+ 8, − 1, − 7	0
3.....	14.65	14.64	14.70	14.66	− 1, − 2, + 4	+ 6
4.....	13.54	13.57	13.71	Bright	.....	.....
5.....	14.87	14.69	14.62	14.73	+14, − 4, −11	+13
6.....	14.77	14.69	14.65	14.70	+ 7, − 1, − 5	+10
7.....	14.73	14.69	14.65	14.69	+ 4, 0, − 4	+ 9
10.....	14.60	14.49	14.51	14.53	+ 7, − 4, − 2	− 7
11.....	14.80	14.72	14.70	14.74	+ 6, − 2, − 4	+14
13.....	14.68	14.69	14.57	14.65	+ 3, + 4, − 8	+ 5
14.....	14.65	14.57	14.61	14.61	+ 4, − 4, 0	+ 1
16.....	14.68	14.66	14.57	14.64	+ 4, + 2, − 7	+ 4
17.....	13.61	14.19	13.92	Bright	.....	.....
18.....	14.34	14.56	14.31	14.40	− 6, +16, − 9	−20
19.....	14.29	Contact‡	14.17	14.23	+ 6, ..., − 6	−37
20.....	13.54	13.78	13.70	Bright	.....	.....
21.....	14.68	14.69	14.57	14.65	+ 3, + 4, − 8	+ 5
23.....	14.77	14.69	14.57	14.68	+ 9, + 1, −11	+ 8
25.....	14.57	14.54	14.57	14.56	+ 1, − 2, + 1	− 4
26.....	14.65	14.72	14.57	14.65	0, + 7, − 8	+ 5
27.....	13.60	14.02	13.99	Bright	.....	.....
28.....	14.33	14.40	14.23	14.32	+ 1, + 8, − 9	−28
29.....	14.73	14.85	14.70	14.76	− 3, + 9, − 6	+16
30.....	14.57	14.72	14.46	14.58	− 1, +14, −12	− 2
31.....	14.73	14.72	14.57	14.67	+ 6, + 5, −10	+ 7
33.....	14.52	14.69	14.40	14.54	− 2, +15, −14	− 6
36.....	14.65	14.69	14.46	14.60	+ 5, + 9, −14	0
39.....	14.73	14.66	14.72	14.70	+ 3, − 4, + 2	+10
40.....	14.73	14.66	14.53	14.64	+ 9, + 2, −11	+ 4
43.....	14.65	14.59	14.49	14.58	+ 7, + 1, − 9	− 2
Mean.....				14.60	Mean.....	±0.09

* Stars fainter than the brightest thirty are omitted.
† Residuals and deviations from mean expressed in hundredths of a magnitude.
‡ Cluster star in contact with a Polar Standard.

For the average cluster the mean value of the photographic magnitude, in the fourth column, depends upon about 500 measures. Its estimated probable error varies from two- to four-tenths of a magnitude, the principal uncertainty coming from possible non-homogeneity of the clusters and from the error in choosing the area to be measured. The average deviation shows the dispersion

of the magnitudes entering the mean, but gives little indication of the certainty of the result; the extremes also show the dispersion, and the upper limit records the highest luminosity of the individual stars.

TABLE IV

MAGNITUDES OF BRIGHT STARS IN MESSIER 53 (N.G.C. 5024)

STAR*	PHOTOGRAPHIC MAGNITUDES			RESIDUALS†	DEVIATION
	2357P	2568P	Means		
2	15.03	14.98	15.00	+ 3, − 2	− 8
4	15.25	15.21	15.23	+ 2, − 2	+15
6	14.99	15.01	15.00	− 1, + 1	− 8
7	14.97	14.93	14.95	+ 2, − 2	−12
9	14.95	15.10	15.02	− 7, + 8	− 6
11	15.17	15.26	15.22	− 5, + 4	+14
12	15.10	15.26	15.18	− 8, + 8	+10
13	15.00	14.93	14.96	+ 4, − 3	−12
15	15.03	14.91	14.97	+ 6, − 6	−11
16	15.03	15.21	15.12	− 9, + 9	+ 4
17	15.03	14.88	Bright	.....	.....
18	14.50	14.57	Bright	.....	.....
20	15.27	15.21	15.24	+ 3, − 3	+16
22	15.15	14.93	15.04	+11, −11	− 4
24	14.87	14.57	Bright	.....	.....
26	15.05	15.01	15.03	+ 2, − 2	− 5
27	15.05	14.83	Bright	.....	.....
28	12.90	13.18	Bright	.....	.....
29	15.28	15.24	15.26	+ 2, − 2	+18
30	14.95	15.10	15.02	− 7, + 8	− 6
31	15.03	15.07	15.05	− 2, + 2	− 3
32	15.30	15.14	15.22	+ 8, − 8	+14
33	15.05	14.93	14.99	+ 6, − 6	− 9
34	15.15	15.33	15.24	− 9, + 9	+16
35	14.94	15.03	14.98	− 4, + 5	−10
36	15.05	15.10	15.08	− 3, + 2	+ 0
37	15.05	15.13	15.09	− 4, + 4	+ 1
38	15.05	15.03	15.04	+ 1, − 1	− 4
42	14.94	15.01	14.98	− 4, + 3	−10
45	14.94	15.04	14.99	− 5, + 5	− 9
Mean			15.08	Mean	±0.09

* Stars fainter than the brightest thirty are omitted.

† Residuals and deviations from mean expressed in hundredths of a magnitude.

Adding the difference between the brighter extreme and the mean magnitude to the adopted absolute value of the latter, we get the maximum brightness in each cluster. Thus we find that the highest photographic luminosity never exceeds magnitude −2.5 (unless some of the excluded five are actually cluster stars), and the maximum usually falls slightly below −2. Contrary to expectation, the

mean magnitude is nearer the brighter extreme in nearly one-third of the clusters, suggesting that among the giants the number of stars does not always increase regularly with decreasing luminosity.

In the seventh column of Table II weights are assigned each cluster on the basis of the quality of the plates, their number, the character of the surrounding stellar field, the certainty of the result for the mean magnitude, and other factors. The remaining columns are described later. Except to remark that the plates for N.G.C. 5139 (ω Centauri) were made with the 10-inch refractor and (because of the very low altitude) are of little value except as a check on Bailey's magnitudes, further space will not be taken for the extensive notes compiled relative to the peculiarities of individual clusters, the observations, the measures, and the investigations of errors.

The equatorial and galactic co-ordinates, the parallaxes, and the co-ordinates in space of the 28 globular clusters[1] for which magnitudes have been measured appear in Table V. For a few clusters the values of the parallax in the sixth column are taken from Table I and for the remainder are computed directly from the mean magnitudes of Table II.

If we plot the parallaxes derived from magnitudes against the diameters of the clusters, as recorded by Melotte,[2] a very definite progression of size with parallax is apparent. Melotte's estimates were made directly from the original Franklin-Adams chart plates; but there is no record of what accuracy was sought or what homogeneity may be expected in the results. Accordingly the diameters have been redetermined from the photographic copies of the plates with the special purpose in view of obtaining results as nearly comparable as possible for all parts of the sky. Measures of the diameters of the images were made independently by two observers, using a finely divided scale under low magnification. By estimating the

[1] N.G.C. 6642 is retained in the table as a twenty-ninth entry. Melotte, with some doubt, classifies it as a globular cluster; Bailey thinks that its stars, few in number, are involved in nebulosity. Mount Wilson plates show a few stars closely crowded, but almost certainly not forming a typical globular cluster. The group is in a rich galactic field.

[2] *Memoirs of the Royal Astronomical Society*, **60**, Part 5, 1915.

TABLE V

PARALLAXES OF 28 GLOBULAR CLUSTERS FROM MAGNITUDES AND DIAMETERS

DESIGNATION		POSITION IN 1900		GALACTIC		PARALLAX (UNIT IS 0".000001)			DISTANCE (UNIT IS 100 PARSECS)		
N.G.C.	Messier	R.A.	Decl.	Long.	Lat.	From Mag.	From Diam.	Adopted	Radial	Projected on Galactic Plane	From Galactic Plane
288	.....	0^h47^m8	−27° 8′	214°	−88°	55	52	53	189	7	−189
1904	79	5 20.1	−24 37	195	−28	44	36	39	256	226	−120
4147	.....	12 5.0	+19 6	227	+78	24	17	19	526	109	+514
5024	53	13 8.0	+18 42	307	+79	48	62	53	189	36	+186
5139	.....	13 20.8	−46 47	277	+16	150	155	153	65	62	+ 18
5272	3	13 37.6	+28 53	8	+77	72	72	72	139	31	+135
5904	5	15 13.5	+ 2 27	333	+45	80	81	80	125	88	+ 88
6093	80	16 11.1	−22 44	320	+18	53	48	50	200	190	+ 62
6121	4	16 17.5	−26 17	319	+15	85	90	88	114	110	+ 30
6205	13	16 38.1	+36 39	26	+40	89	91	90	111	85	+ 71
6218	12	16 42.0	− 1 46	344	+25	80	82	81	123	111	+ 52
6229	.....	16 44.2	+47 42	41	+39	29	20	23	435	338	+274
6254	10	16 51.9	− 3 57	343	+22	77	89	83	120	111	+ 45
6333	9	17 13.3	−18 25	334	+ 9	38	44	40	250	247	+ 39
6341	92	17 14.1	+43 15	35	+34	84	78	81	123	102	+ 69
6356	.....	17 17.8	−17 43	335	+ 9	19	30	26	385	380	+ 60
6402	14	17 32.4	− 3 11	349	+14	41	46	43	233	226	+ 56
6626	28	18 18.4	−24 55	336	− 7	53	57	54	185	184	− 23
6638	.....	18 24.8	−25 34	335	− 8	29	28	29	345	342	− 48
6642*	.....	18 25.8	−23 32	337	− 7	31	22	26	385	382	− 47
6656	22	18 30.3	−23 59	338	− 9	118	116	118	85	84	− 13
6712	.....	18 47.6	− 8 50	353	− 6	30	33	32	312	310	− 32
6779	56	19 12.7	+30 0	30	+ 7	43	37	40	250	248	+ 30
6864	75	20 0.2	−22 12	348	−28	19	26	22	455	401	−213
6934	.....	20 29.3	+ 7 4	20	−20	35	24	30	333	313	−114
6981	72	20 48.0	−12 55	3	−34	33	35	34	294	244	−164
7078	15	21 25.2	+11 44	33	−29	68	59	68	147	129	− 71
7089	2	21 28.3	− 1 16	22	−37	60	72	64	156	125	− 94
7099	30	21 34.7	−23 38	356	−48	59	56	58	172	115	−128

*See n. 1 on p. 160.

elongation of similarly situated bright stars corrections have been made for distortion when a cluster is near the edge of the chart;[1] and to counteract the frequently observed actual ellipticity of figure the mean of the diameters in several directions has been determined for all clusters. The last three columns of Table II contain these closely agreeing measures and their mean, thus affording a fairly homogeneous record of relative apparent dimensions. The diameters estimated from the original plates are usually larger, as would be expected, but even they fall short of the actual dimensions for many clusters.

Plotting the adopted diameters of the last column of Table II against the parallaxes derived from magnitudes in Table V, we obtain the curve in Fig. 1—a curve that demands and apparently justifies the hypothesis that all globular clusters are of nearly the same linear dimensions. There is no reason for supposing that the more distant clusters are actually and systematically smaller than nearer ones; hence, as is reasonable a priori, the non-linear form of this curve may be attributed to the characteristic distribution of luminosity in a cluster and its resulting effect on photographic reproduction. Normal points for the data underlying the figure are given in Table VI and are plotted as black circles.[2] The lower part of the curve is somewhat uncertain, depending only on Messier 22 and ω Centauri, but the uncertainty is not important, for only two or three clusters are large enough to make their parallaxes depend on that part of the curve.

Undoubtedly an improvement can be made in at least some of the parallaxes through the process of smoothing with the aid of the

[1] Many of the clusters appear on two or more charts. The differences in quality from plate to plate, which may be greater for the charts, are not nearly so effective in measures of angular diameter as they would be in estimates of magnitudes. Fortunately about 90 per cent of the clusters are on the Johannesburg plates, which attain a fainter limit of magnitude and are more uniform than those made for the northern sky at Mervel Hill (*Memoirs of the Royal Astronomical Society*, **60**, 167, 1914).

[2] Half weight is given to N.G.C. 6642; also to N.G.C. 6712 because of a slight doubt as to its nature and because it is in such a rich region of the galactic clouds that the measured diameter is a little uncertain. Cf. *Publications of the Astronomical Society of the Pacific*, **29**, 186, 1917. Possibly a doubt should also be expressed as to the perfectly normal nature of N.G.C. 4147. Cf. the eleventh paper of this series.

parallax-diameter curve of Fig. 1. Accordingly, parallaxes corresponding to the measured diameters are entered in the seventh column of Table V and are combined with the parallaxes from magnitudes to obtain the smoothed values of the eighth column. The combination is made with due regard for the relative quality of the magnitude work, assigning for convenience weight unity

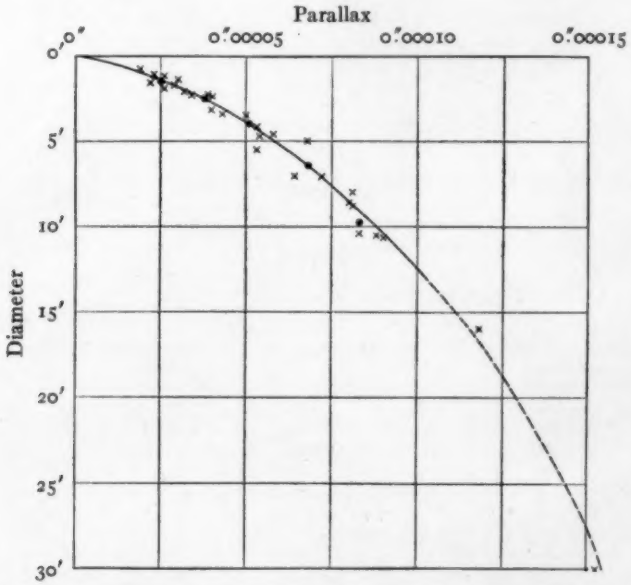


FIG. 1.—The parallax-diameter curve for globular clusters (diameters from Franklin-Adams charts). Dots are normals based upon parallaxes from magnitudes alone; crosses are the finally adopted values for the 29 clusters of Table V.

to all the parallaxes from magnitudes, and, to the corresponding values from angular measurements, the weight zero for group *a*, one-half for group *b*, one for group *c*, and two for group *d*. The adopted parallaxes are plotted in Fig. 1 as crosses.

Before obtaining the parallaxes of other clusters from their diameters alone, we shall note what accuracy may be expected in the results. In Table VII are given the percentage deviations from the parallax-diameter curve, both for the original parallaxes from magnitudes and for the adopted values. Without assigning

weights, the arithmetical[1] mean for the first is 14 per cent, and for the second, 7 per cent. The latter shows the average amount of the discrepancy that will affect the parallaxes of the clusters for which no magnitudes are available. The deviations are partly

TABLE VI

DIAMETERS AND PARALLAXES

Number of Clusters	Mean Parallax	Mean Diameter
7	0".000025	1'.5
5	0.000038	2.5
5	0.000051	4.0
5	0.000068	6.4
5	0.000083	9.7
1	0.000118	16
1	0.000150	30

TABLE VII

DEVIATIONS FROM THE PARALLAX-DIAMETER CURVE

N.G.C.	$\frac{\pi \text{ Mag.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$	$\frac{\pi \text{ Adopt.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$	N.G.C.	$\frac{\pi \text{ Mag.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$	$\frac{\pi \text{ Adopt.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$
288	+0.06	+0.02	6356	−0.42	−0.15
1904	+ 20	+ 8	6402	− 12	− 7
4147	+ 37	+ 11	6626	− 7	− 6
5024	− 26	− 17	6638	+ 3	+ 3
5139	− 3	− 1	6656	+ 2	+ 2
5272	0	0	6712	− 9	− 3
5904	− 1	− 1	6779	+ 15	+ 8
6093	+ 10	+ 4	6864	− 32	− 18
6121	− 6	− 2	6934	+ 37	+ 20
6205	− 2	− 1	6981	− 6	− 3
6218	− 2	− 1	7078	+ 13	+ 13
6229	+ 39	+ 13	7089	+ 19	− 12
6254	− 14	− 7	7099	+0.05	+0.03
6333	−0.15	−0.10			

due to real differences in the clusters, but most of the error is within the uncertainty of angular measurement, for the average difference between the estimates of diameter by the two observers is but slightly less than 10 per cent.

Nearly all of the 41 clusters included in Table VIII are south of declination −30°. The few exceptions will be photographed with the 60-inch reflector, when opportunity permits, and the

[1] For brevity the terms "algebraic mean" and "arithmetical mean" are used in the sense of *with* and *without* regard to sign.

TABLE VIII

PARALLAXES OF 41 GLOBULAR CLUSTERS FROM DIAMETERS

N.G.C.	R.A. 1900	DECL. 1900	GALACTIC		ANGULAR DIAMETER *D*, *S*	PARALLAX (UNIT IS 0".000001)	DISTANCE (UNIT IS 100 PARSECS)		
			Long.	Lat.			Radial	Projected on Galactic Plane	From Galactic Plane
104	$0^h 19^m.6$	$-72° 38'$	272°	−44°	26'.0, 27'.6	148	68	49	− 47
362	0 58.9	−71 23	268	−46	5.4, 6.6	66	152	106	−109
1261	3 9.5	−55 36	238	−51	2.6, 2.5	39	256	161	−199
1851	5 10.8	−40 9	211	−34	4.8, 5.0	58	172	143	− 96
2298	6 45.4	−35 54	213	−15	3.0, 2.7	41	244	236	− 63
2808	9 10.0	−64 27	249	−11	5.2, 4.9	59	170	167	− 32
3201	10 13.5	−45 54	244	+10	6.6, 6.3	68	147	145	+ 26
4372	12 20.1	−72 7	269	− 9	10.8, 9.3	88	114	113	− 18
4590	12 34.2	−26 12	268	+37	4.8, 5.8	62	161	129	+ 97
4833	12 52.7	−70 20	271	− 8	5.6, 4.9	61	164	163	− 23
5286	13 40.1	−50 52	279	+11	3.6, 4.5	51	196	193	+ 37
5634	14 24.4	− 5 32	309	+49	2.2, 2.0	33	303	199	+229
5897	15 11.7	−20 39	311	+30	6.4, 6.2	67	149	129	+ 75
5986	15 39.5	−37 27	305	+13	3.3, 4.1	48	208	203	+ 47
6101	16 14.4	−71 58	284	−15	3.0, 3.7	47	213	206	− 55
6144	16 21.2	−25 49	319	+15	2.8, 2.8	41	244	236	+ 63
6171	16 26.9	−12 50	331	+22	5.8, 5.0	62	161	149	+ 60
6235	16 47.4	−22 1	326	+13	1.0, 1.2	20	500	487	+112
6266	16 54.8	−29 58	320	+ 7	6.4, 5.9	66	152	151	+ 19
6273	16 56.4	−26 7	324	+ 9	5.6, 5.4	63	159	157	+ 25
6284	16 58.4	−24 37	325	+10	1.4, 1.9	27	370	364	+ 64
6287	16 59.1	−22 34	327	+11	1.3, 1.4	23	435	428	+ 83
6293	17 4.0	−26 26	325	+ 8	2.9, 2.2	38	263	261	+ 37
6304	17 8.2	−29 20	323	+ 5	2.0, 1.9	31	322	320	+ 28
6316	17 10.3	−28 1	325	+ 5	1.0, 1.0	19	526	524	+ 46
6352	17 17.8	−48 19	308	− 7	3.4, 3.0	44	227	226	− 28
6362	17 21.5	−66 58	293	−17	7.3, 8.4	77	130	124	− 38

6388	17 29.0	−44 40	312	− 7	2.5, 2.2	36	278	276	− 34
6397	17 32.5	−53 37	304	−12	18.0, 16.2	120	83	81	− 17
6441	17 43.4	−37 1	321	− 5	1.3, 1.2	22	455	453	− 40
6541	18 0.8	−43 44	316	−11	6.8, 6.2	68	147	144	− 28
6584	18 10.6	−52 15	309	−16	2.4, 2.6	38	263	253	− 73
6624	18 17.3	−30 24	330	− 8	2.4, 2.2	35	286	283	− 40
6637	18 24.8	−32 25	329	−11	3.3, 3.6	47	213	209	− 41
6652	18 29.2	−33 4	328	−12	2.0, 2.0	32	312	305	− 65
6681	18 36.7	−32 23	329	−13	4.4, 4.6	55	182	177	− 41
6715	18 48.7	−30 36	333	−16	5.7, 5.2	62	161	155	− 44
6723	18 52.8	−36 46	327	−18	7.3, 9.2	79	127	121	− 39
6752	19 2.0	−60 8	303	−26	16.8, 14.4	114	88	79	− 39
6809	19 33.7	−31 10	335	−24	11.7, 13.2	100	100	91	− 41
7006	20 56.8	+15 48	32	−20	0.8, 0.7	15	667	626	−228

parallax, as determined from diameters, will receive an independent check from the study of magnitudes.[1] The right ascension, declination, and galactic co-ordinates in Table VIII, as in previous tables, have been taken, whenever possible, from Melotte's catalogue, the results being checked with Bailey's lists and other catalogues. The angular diameters in the fifth column were obtained at the same time as those measured for Table V; their mean value is the basis of the parallaxes in the sixth column. The first cluster of this list is 47 Tucanae.

Tables V and VIII contain all clusters now thought to be definitely globular—a total of 69. The 15 or 20 others[2] frequently

[1] Since writing the above, one highly satisfactory confirmation has been secured. The object N.G.C. 7006 was noted as a small faint cluster by Curtis (*Lick Observatory Bulletins*, **7**, 84, 1912). On the Franklin-Adams plates and charts it appears as a nebulous star and was excluded from Melotte's catalogue of globular clusters. It is not mentioned by Bailey. The apparent diameter as given in Table VIII is only 0'.75—the smallest cluster of the two lists. If our hypotheses are right, therefore, it should be the most distant cluster, and the mean magnitude should be fainter than any hitherto measured. Two polar-comparison photographs were secured in December 1917. A seven-minute exposure on a fast plate shows about two hundred stars brighter than magnitude 18.5, and one star near the center is nearly as bright as magnitude 15; but the mean of the 25 brightest, according to the preliminary measures, is 17.7. The corresponding parallax is a little more than 0".000014, differing by less than a millionth of a second from the value in Table VIII.

[2] Among the clusters thought by some to be globular are the following:

N.G.C.	Angular Diameter	Galactic		Remarks
		Long.	Lat.	
371.....	2'.1	268°	−44°	In Small Mag. Cloud. Not typical, Bailey. If globular, it gives for the distance of the cloud $\pi = 0''.000033$. Cf. Table I.
2660.....	1.5	234	− 2	Appears to be globular cluster, Melotte. If so, $R \sin \beta = -1400$ parsecs; $R \cos \beta = 39{,}700$ parsecs.
5466.....	5.4	10	+72	Apparently an open cluster, Shapley. If globular, $\pi = 0''.00006$.
6496.....	1.5	315	−11	Probably a nebula. Bailey does not mention it.
6535.....	0.8	354	+10	A group of a few faint stars, Shapley.
6569.....	1.3	328	− 7	A nebula according to Bailey. If a globular cluster, $\pi = 0''.00002$.
6760.....	1.0	3	− 5	Small cluster of very faint stars, Curtis, Pease. An open cluster, Shapley.
7402.....	3.5	21	−64	An open cluster, Curtis, Shapley. Very loose globular cluster, Melotte. If globular, $\pi = 0''.00005$.

Bibliography: Bailey, *Harvard Annals*, **60**, No. 8, 1908; **76**, No. 4, 1915. Curtis, *Lick Observatory Bulletins*, **7**, 81, 1912; **8**, 43, 1913. Melotte, *Memoirs of the Royal Astronomical Society*, **60**, Part V, 1915. Pease, *Publications of the Astronomical Society of the Pacific*, **26**, 204, 1914. Shapley, *Publications of the Astronomical Society of the Pacific*, **29**, 186, 1917. Shaw, *Helwan Observatory Bulletins*, No. 9, 1912; No. 15, 1915; *Monthly Notices*, **76**, 105, 1915.

admitted to the catalogues are rejected temporarily on the basis of either the Mount Wilson plates or the published opinions of Bailey, Curtis, Shaw, or Melotte. Some questioned groups may be admitted later, and other faint objects now considered nebulous stars or open clusters or faint nebulae may be proved by the large reflectors to be globular clusters. But as far as systems containing stars brighter than the sixteenth photographic magnitude are concerned, the present work may be considered exhaustive.

II. DISTRIBUTION IN SPACE

Some striking features of the arrangement of clusters in space are brought to light by a study of the data in Tables V and VIII. Fig. 2 illustrates the distribution in three dimensions, showing on the plane of the Milky Way the galactic longitudes and projected distances, while the distances from the Galaxy are shown by vectors drawn to scale in the plane of the figure. Distances above (north of) the galactic plane are represented by full heavy lines drawn upward from black circular bases, those below by broken lines downward from open circular bases. To visualize the actual positions in space one needs only to imagine the full-line vectors standing erect on their bases and the broken lines hanging vertically from theirs; the arrow points are then at the positions of the clusters.

The most remote of all the clusters[1] is N.G.C. 7006 with a distance of 67,000 parsecs, the equivalent of 220,000 light-years; the clusters N.G.C. 4147, 6229, 6235, 6287, 6441, and 6864 (M 75) are nearly as far away. Fortunately, of these seven most distant systems six are within reach of the Mount Wilson reflectors. One-fourth of all globular clusters appear to be more distant than 30,000 parsecs (100,000 light-years); and one (N.G.C. 4147) is more than 50,000 parsecs from the plane of the Milky Way. ω Centauri and 47 Tucanae, with distances somewhat less than 7000 parsecs, are the clusters nearest to the sun.

The concentration into a limited interval of galactic longitude is conspicuous; the region from 41° to 195° is completely void of globular clusters. The mean value of all longitudes is 316°, or,

[1] See n. 1, p. 167, of this paper and the later discussion in the eleventh paper.

excluding the five largest and five smallest values, is 318°; but the mean is too much affected by widely divergent values, and the

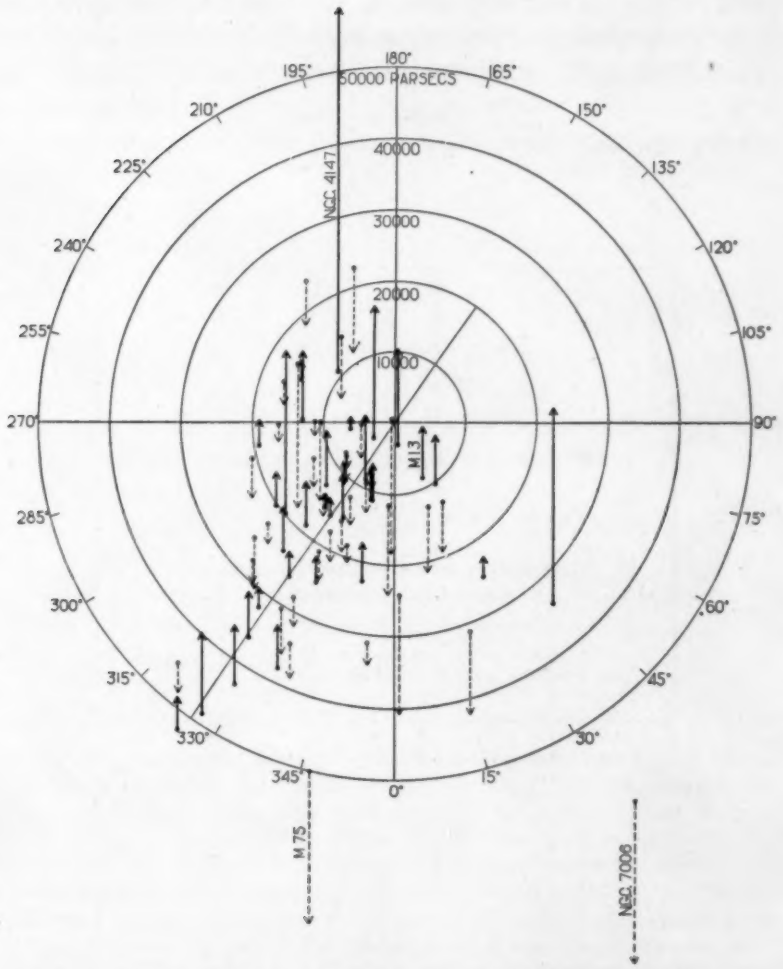


FIG. 2.—Distribution in space of globular clusters. The galactic plane is the plane of the diagram; distances above and below are shown to scale by full-line and broken-line vectors, respectively. Galactic longitudes are indicated in the margin and the scale of distances along the vertical radius. The sun is at the origin of co-ordinates. The diagram illustrates the remarkable distribution in longitude, with a maximum frequency at 325°, and by the absence of very small or zero vectors shows that globular clusters are not found within 1000 parsecs of the plane of the Milky Way. Cf. Fig. 1 of the twelfth paper.

median value,[1] 325°, is preferable in determining the central line of the system of globular clusters. The frequency of longitudes shown by Fig. 3 agrees in placing the center in longitude 325°, the points for the curve depending on the data of Table IX.

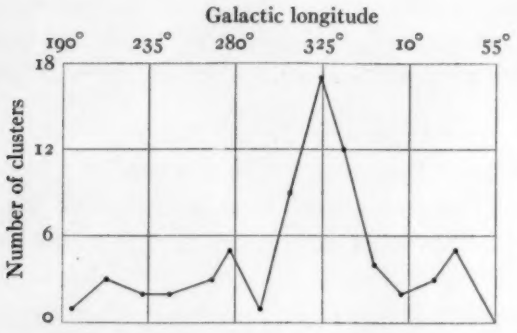


FIG. 3—Distribution of globular clusters in galactic longitude

TABLE IX

DISTRIBUTION OF GLOBULAR CLUSTERS IN GALACTIC LONGITUDE

Interval of Longitude	Number of Clusters	Mean Longitude
195° to 210°	1	195°
210 to 225	3	213
225 to 240	2	232
240 to 255	2	246
255 to 270	3	268
270 to 285	5	277
285 to 300	1	293
300 to 315	9	308
315 to 330	17	324
330 to 345	12	336
345 to 360	4	352
0 to 15	2	6
15 to 30	3	23
30 to 45	5	34

Projecting all positions on to a plane through the sun perpendicular to the Galaxy and including the circle defined by galactic

[1] That is, the longitude of the thirty-fifth cluster when they are taken in order of increasing longitude, beginning with N.G.C. 1904.

longitude 325°, we get the diagram in Fig. 4, which represents the distribution as seen from a great distance in the direction of galactic latitude 0° and galactic longitude 235°. Black dots above the central line represent clusters north of the galactic plane; open circles below represent those south. Thus the ordinates are $R \sin \beta$ and the abscissae $R \cos \beta \cos (\lambda - 325°)$, where R, β, and λ are respectively the distance, galactic latitude, and galactic longitude of a cluster.

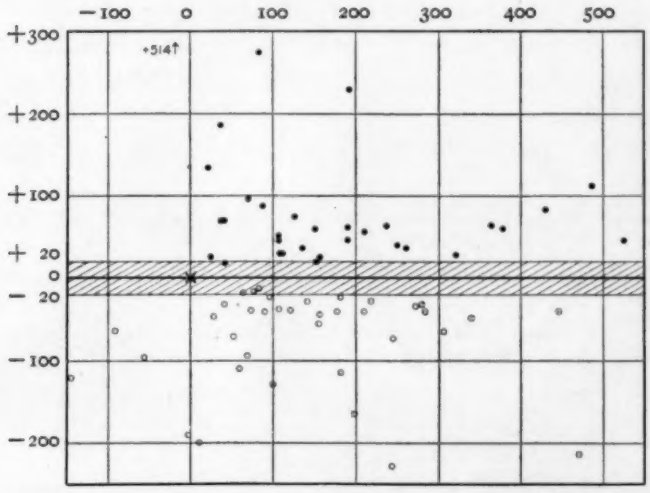


FIG. 4.—Projection of the positions of globular clusters on a plane perpendicular to the Galaxy, illustrating (1) the absence of clusters from the mid-galactic region, (2) their symmetrical arrangement with respect to the Galaxy, (3) the eccentric position of the sun (the cross) with respect to the center of the system of clusters. The ordinates are distances from the galactic plane, $R \sin \beta$; the abscissae are projected distances in the direction of the center, $R \cos \beta \cos (\lambda - 325°)$. The unit of distance is 100 parsecs; the side of a square is accordingly 10,000 parsecs. On this scale the actual diameter of the clusters is about one-fifth the diameter of the circles and dots. The cluster N.G.C. 4147 is outside the boundary of the diagram, as indicated by the arrow.

Fig. 5 differs from the preceding diagram only in having $R \cos \beta$ for abscissae. Hence the sun, as origin, is at the extreme left edge of the figure, and the actual distance of each cluster is represented by the radial distance from the origin.

Figs. 4 and 5 show more clearly than Fig. 2 the remarkable distribution of clusters with respect to the Galaxy. In the first place clusters are impartially distributed above and below, there being 32 north and 37 south of the plane. Although the average value without regard to sign is ±79 (in units of 100 parsecs), the algebraic mean of all distances from the plane is −1; rejection of

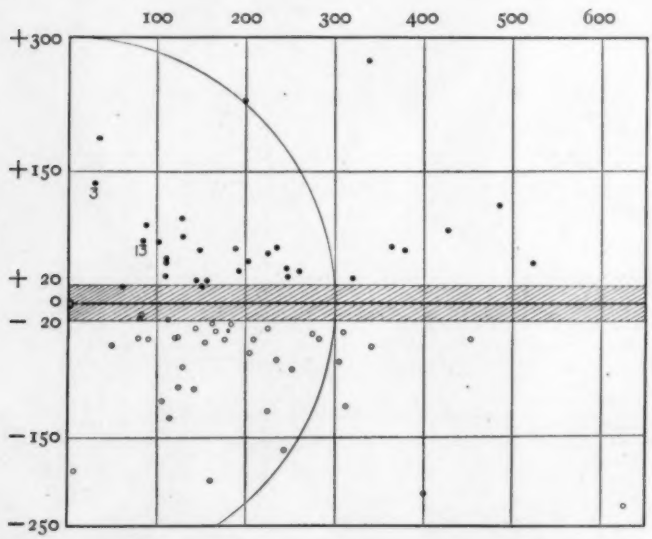


FIG. 5.—Distribution of globular clusters. The ordinates are $R \sin \beta$, as in Fig. 4; the abscissae are distances projected on the galactic plane, $R \cos \beta$. The unit of distance is 100 parsecs. The very small semicircle, with radius corresponding to a parallax of 0".002, illustrates the region around the sun which contains all but a few of the stars in Charlier's B-type cluster. The large semicircle indicates the distance to which the present results are thought to be complete. Messier 3 and Messier 13 are indicated by numbers; the most distant cluster now known, N.G.C. 7006, is near the lower right-hand corner of the diagram. N.G.C. 4147, with co-ordinates 109 and 514, is not shown.

the very distant cluster, N.G.C. 7006, would change this to +2. Considering the accidental variation and the size of the distances involved, the algebraic mean is vanishingly small, and we may say confidently that the plane of the Milky Way is also a symmetrical plane in the great system of globular clusters. This relation to the Galaxy holds with good approximation at all distances from the

sun (graphically shown best by Fig. 5), as may be seen from the following tabulation:

	Interval of $R \cos \beta$				
	0–100	100–200	200–300	>300	All
Number of Clusters	11	29	15	14	69
Mean* $R \sin \beta$ Algebraic	+ 7	+ 9	−23	− 5	− 1
Mean* $R \sin \beta$ Arithmetical	±70	±84	±59	±100	±79

*See n. 1, p. 164.

A second phenomenon clearly illustrated by the diagrams is the avoidance of the Milky Way—a result that may be of very exceptional significance. There is no cluster within 1300 parsecs of the plane of the Galaxy, and within 2000 parsecs of that plane there are only five, four of which are among the clusters nearest

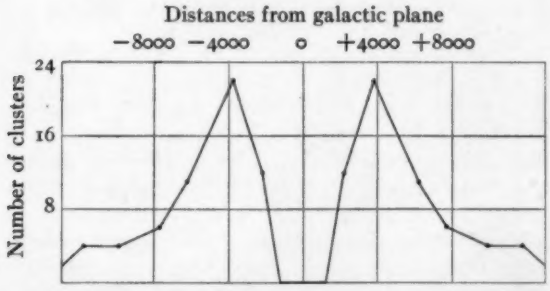


FIG. 6.—Reflected frequency-curve of the distances of globular clusters from the galactic plane, illustrating the equatorial region devoid of globular clusters. The unit of distance is one parsec.

the sun. In Figs. 4 and 5 a shaded region, 13,000 light-years in width, indicates the zone from which globular clusters are practically excluded.

The increasing concentration toward the Galaxy from both sides stops almost abruptly at the boundary of the shaded zone. The frequency of distances from the plane is treated in more detail in Table X, and the undoubted dependence of the clusters on the galactic plane, noted numerically above, is further emphasized by the curve in Fig. 6. The completion of that curve, in a form naturally to be expected for the frequency of objects concentrated toward the

TABLE X

THE FREQUENCY OF DISTANCES FROM THE GALACTIC PLANE ($R \sin \beta$)

LIMITS OF $R \sin \beta$ (UNIT IS 100 PARSECS)	$R \cos \beta < 200$					$R \cos \beta > 200$					ALL CLUSTERS	
	No. of Clusters		Mean $R \sin \beta$			No. of Clusters		Mean $R \sin \beta$				
	North	South	North	South	All	North	South	North	South	All	Number	Mean $R \sin \beta$
<10	0	0	.....	.....	.....	0	0	.....	.....	.....	0	.....
10– 30	4	6	22	20	21	1	1	28	28	28	12	22
30– 50	3	8	37	40	39	5	6	40	39	39	22	39
50– 70	4	0	61	.....	61	4	3	61	61	61	11	61
70– 90	3	1	78	71	76	1	1	83	73	78	6	77
90–110	1	3	97	100	99	0	0	.....	.....	.....	4	99
110–130	0	1	.....	128	128	1	2	112	117	119	4	118
130–150	1	0	135	.....	135	0	0	.....	.....	.....	1	135
150–170	0	0	.....	.....	.....	0	1	.....	164	164	1	164
170–190	1	1	186	189	188	0	0	.....	.....	.....	2	188
190–210	0	1	.....	199	199	0	0	.....	.....	.....	1	199
210–230	1	0	229	.....	229	0	2	.....	220	220	3	223
>230	1	0	514	.....	514	1	0	274	.....	274	2	394
Totals	19	21	.....	.....	.....	13	16	.....	.....	.....	69	.....

Galaxy, would require at least 50 globular clusters within 1500 parsecs of the plane; there is, however, only one, Messier 22, and its distance below the plane corresponds to a parallax of 0".0008. It should be observed, moreover, that not only at great distances, where insufficiency of observations may be urged, do we note this absence of clusters, but also within a distance from the sun of 20,000 parsecs, where the data are quite sufficient (37 clusters) and undoubtedly are complete. Hence we conclude that this great mid-galactic region, which is peculiarly rich in all types of stars, planetary nebulae, and open clusters, is unquestionably a region unoccupied by globular clusters.

To explain this remarkable condition several hypotheses have been considered, such as error in choosing the origin of galactic latitudes,[1] incompleteness of data, general absorption of light in space,[2] clouds or a ring of absorbing matter along the spine of the Milky Way analogous to the dark peripheral rings of spiral nebulae,[3]

[1] The adopted position of the north galactic pole is that given by Gould (*Uranometria Argentina*): $\alpha = 12^h41^m$, $\delta = +27°21'$. This position differs by less than a degree from those obtained through other reliable and definitive investigations. The most recent, and probably the best, is based on the Harvard Map of the Sky, by Nort, who finds $\alpha = 12^h44^m$, $\delta = +27°$ (*Recherches Astronomiques de l'Observatoire d'Utrecht*, VII, 84, 112, 1917). An error of $\pm 1°$ in the cluster latitudes might slightly displace, widen, or narrow the zone of avoidance, but nothing short of selectively operative errors of several degrees could seriously obscure it. The frequency-curve of galactic latitudes also shows the zone, but naturally to a less degree. Cf. Fig. 2 of the twelfth paper. The values in Tables V and VIII show no latitude less than 5°. See the remarks relative to N.G.C. 2660 in n. 2, p. 167.

[2] See the notes in a following paper (the eleventh of the series) on the color of stars in the two most distant clusters. The dark obstructing nebulae which are frequently found in and near the Milky Way are undoubtedly capable of obliterating or greatly diminishing the light of any cluster involved in the nebulosity or beyond it. N.G.C. 4372, a large southern cluster (almost certainly globular), which is very faint for its angular diameter, falls alongside a vacant space in the sky. N.G.C. 6144 is near the edge of the ρ Ophiuchi dark nebulosity and appears large for the magnitude of its bright stars. This latter nebulosity may also affect the brightness of Messier 4 (N.G.C. 6121) to some extent. But it is interesting to note that in developing a parallax method that is independent of the magnitudes we have escaped from the errors in parallax that such obstructing material might occasionally have imposed.

[3] Possibly a hypothetical wedge-shaped ring might explain some of the divergence from the galactic plane with increasing distance (Figs. 4 and 5); but insufficiency of material for faint clusters would better account for most of it. The phenomenon, of course, may be real—a widening of the zone of avoidance in the direction of the

and finally the actual absence of globular clusters from the regions rich in stars because of the dynamical impossibility of existence. The first three seem clearly impossible, the fourth improbable or at least unquestionably insufficient, and, therefore, without going at present into the meaning and consequences of such a theory, the last hypothesis is tentatively adopted.

We have found that the center of the elongated and somewhat irregular system of globular clusters lies in the plane of the Milky Way on a line directed toward galactic longitude 325°. The distance along that line may be estimated from an inspection of Fig. 4, and perhaps obtained more accurately from the following consideration of the frequencies of $R\cos\beta\cos(\lambda-325°)$, in which clusters more than 15,000 parsecs distant from the plane are excluded:

	Interval of Distance in Direction of Center								
	<100	−100 to 0	0 to +100	+100 to +200	+200 to +300	+300 to +400	+400 to +500	>500	All
Number of clusters	1	2	18	20	10	5	3	1	60
Mean of $R\cos\beta\cos(\lambda-325°)$...	−145	−74	+59	+138	+246	+342	+454	+525	+158

From a plot of these numbers we estimate provisionally the distance of the center to be 13,000 parsecs. Incompleteness of data because of faintness will not materially affect the galactic longitude and latitude of this point, but is likely to make the distance too small. The mean value of $R\cos\beta\cos(\lambda-325°)$ for the 60 clusters, +158, is probably nearer the true value, but it is also liable to understate the distance. A definitive value is hardly possible, and, at least until the number of very faint clusters can be considerably increased, we may adopt as the center of the general system of globular clusters a point for which the parallax is between 0".00006 and 0".00004, with equatorial co-ordinates $\alpha=17^h30^m$, $\delta=-30°$, and galactic co-ordinates $\lambda=325°$, $\beta=0°$. The position

center of the globular cluster system, combined with a lack of observations for clusters far beyond the center. See n. 2, p. 167; if N.G.C. 2660 is globular, the apparent tendency to widen with distance is somewhat counteracted.

lies in the constellation Sagittarius, a few degrees east of its boundary with Scorpio and Ophiuchus.

III. LINEAR DIMENSIONS OF CLUSTERS

A literal interpretation of the curve in Fig. 1 gives for the dimensions of globular clusters:

	Parallax (Unit Is 0".000001)						
	20	40	60	80	100	120	150
Angular diameter	1'.1	2'.7	5'.2	8'.3	12'.4	17'.2	27'.5
Linear diameter in parsecs.....	16	20	25	30	36	42	53

As remarked before, however, the diameters from the charts do not give a true measure of the clusters, and the hypothesis that size depends on distance from the sun, irrespective of distance from the Galaxy, is quite untenable.

There are, without doubt, real though relatively inconspicuous differences among globular clusters; and some of the differences, such as frequency of certain spectral types, degree of condensation and ellipticity, and possibly total numbers of stars, are being recognized and evaluated through the Mount Wilson studies. Two properties, however, so far seem to show little variation from cluster to cluster—the actual linear diameter and the mean magnitude of the brightest stars. We shall assume, therefore, on the basis of what observational evidence we now have at hand, that all globular clusters are of practically the same dimensions, explaining the apparent decrease in size with increasing distance (tabulated above) as a natural consequence of a central concentration of luminosity and of an intermingling near the edges with non-cluster stars. On that basis an investigation of the dimensions of one cluster will suffice for all.

Long exposures with the 60-inch reflector upon the outer parts of some of the brightest clusters have confirmed the conclusion, obtained from a study of the distribution of variable stars, that the clusters are much greater in extent than would be inferred from ordinary visual or photographic observation. For instance, on the

Franklin-Adams charts the apparent diameter[1] of Messier 3, the cluster chosen for the present illustration of dimensions, is 7′; on the original plates it is 18′, according to Melotte; but the actual diameter is in excess of half a degree.

The distance of Messier 3 is 13,900 parsecs,[2] corresponding to nearly three thousand million times the distance of the sun from the earth. The distance north of the galactic plane is 13,500 parsecs. The accompanying plate is reproduced from a photograph of several hours' exposure made by Mr. Ritchey with the 60-inch reflector. The original negative shows more than twenty thousand stars outside the central burned-out area, the smallest images being fainter than the twentieth magnitude.

The cluster extends beyond the limits of the photograph in all directions. The most distant variable star (undoubtedly a member of the system) is 17′ from the center, corresponding to a projected distance of fourteen million astronomical units. As ordinarily seen and photographed, the cluster covers an area but little larger than one of the squares, but we may be sure that its actual projected area is at least twenty-five times as great; that is, the diameter is about thirty million astronomical units. To cross the cluster, light must travel 470 years.

If we suppose the sun situated at the center of the cluster, all stars with parallaxes greater than 0″.1 would be included within the concentric circle. Sirius would be at the distance indicated by the cross, and the projected distance of the bright triplet near the bottom of the picture equals the distance of the Hyades from the sun.

Inclosed in small circles are a few of the variable stars, chosen at random, the close equality of whose magnitudes is to be noted. In some cases they appear as doubles, but the actual separation

[1] The estimates from the charts, listed in Tables V and VIII, refer actually to what appears to be a central core of each system. The scale of the photographs does not permit close differentiation of the outlying members of a cluster from the stars of its surrounding field.

[2] The diagram was made on the basis of a parallax of 0″.000074; the final value of Tables I and V indicates that the linear dimensions of the cluster on the plate are too small by 3 per cent, an amount, however, that is far within the probable error.

in the closest of the indicated pairs is more than half the distance separating the sun from α Centauri. In the small square is the image of an exceptional variable, No. 37, for which the period is less than eight hours.

The cluster variables, in the mean, have the absolute magnitude −0.2, photographically nearly six magnitudes brighter than the sun. A star of the brightness and color of the sun would not appear on the photograph, being nearly two magnitudes too faint (21.5). Sirius, located in this cluster, would be of the seventeenth apparent magnitude, corresponding to the star indicated on the photograph by an arrow-point.

The condensation of stars at the center of the cluster may be readily contrasted with that of stars around the sun. Within the circle, which marks a distance from the center corresponding to a parallax of 0".1 (approximately two million astronomical units), there are at least 15,000 stars brighter than magnitude 20. (This estimate deducts those stars not within the concentric sphere but appearing by projection within the circular area.) In a sphere of the same radius, with the sun as center, less than twenty stars brighter than the sun are known. But only those which are two magnitudes brighter appear on this photograph of the cluster; there are, accordingly, in the sphere around the sun only four or five stars to compare with the 15,000 in Messier 3.

Finally, we shall make some estimates relative to the probable mass of a globular cluster. We may go astray, to be sure, in assuming similar masses and analogous relations of mass to luminosity for stars in clusters and in the general galactic system. The dynamical conditions in these highly condensed globular systems may conceivably have some important effect upon the amount of matter that goes into a single star as well as upon the speed and nature of subsequent development. Eddington's recent theoretical work on the masses of stars, however, indicates that as long as we deal with typical giants the masses are definitely limited; and, further, the identity of Cepheid phenomena wherever studied—in the galactic system, in the more condensed Magellanic clouds, and in the extremely condensed globular clusters—tends to support the view of the universal comparability of stellar masses.

All that we know of the masses of stars from observation has been summarized recently by Russell.[1] The data come altogether from double stars, but we shall probably not commit serious error in applying the results without alteration to the single stars in the condensed globular clusters, although possibly slightly greater average values for each type would be appropriate for the isolated stars in our general galactic system. For the present approximation we shall simply take a mean value from Russell's data, say an average of four times the sun's mass for every star brighter than the sun. The number of such stars in Messier 3 may be fairly estimated at 40,000, only three-fourths of which have ever been photographed. Hence the entire mass, distributed among the stars which are brighter absolutely than photographic magnitude +5.6, is 160,000 times the solar mass; and something like three-fourths of this amount is within 10 parsecs of the center.

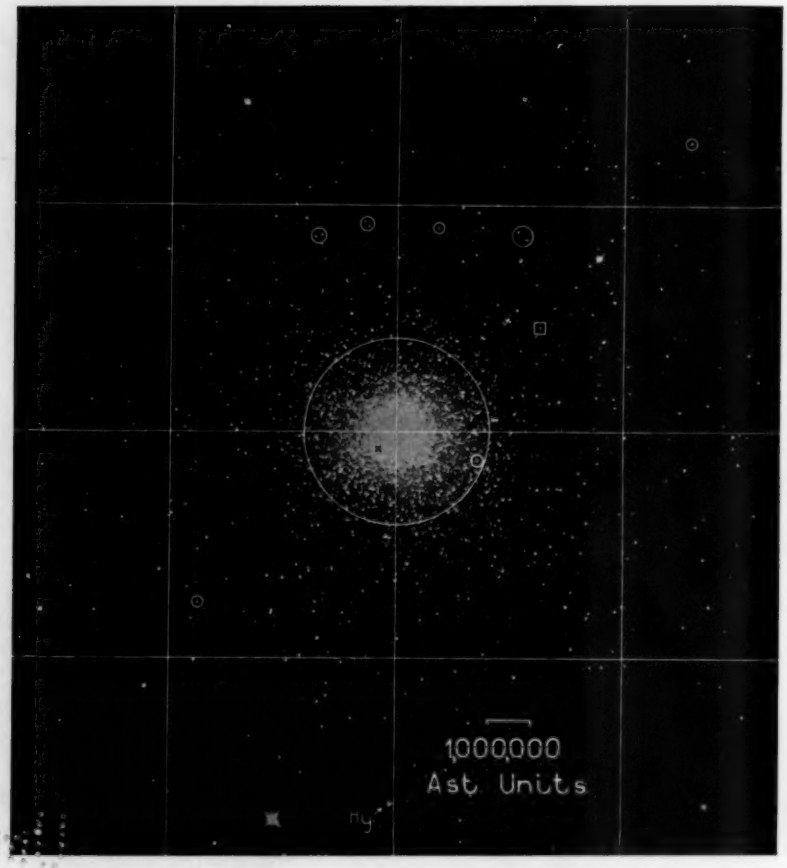
It seems quite futile at present to estimate the total mass of the cluster. Perhaps it is two or three million times the solar mass; probably it is several times the amount estimated for the stars brighter than 5.6—that we must admit on the basis of what little we now know of the relative frequency of dwarfs and giants in the vicinity of the sun; but as a matter of absolute certainty we do not know that there is a single star fainter than the sun, and even our estimate of 160,000 may be 50 per cent too great.

SUMMARY

1. Following the methods outlined in the preceding paper the parallaxes and positions in space are obtained for 69 globular clusters—all that can now be definitely assigned to the globular class. The distances range from 6500 to 67,000 parsecs, the brightest stars in the most distant clusters being fainter than the seventeenth photographic magnitude. The average probable error of a parallax is of the order of 20 per cent, varying considerably with the method used and with the quality of the observational work. Something more than 15,000 measures of magnitudes were made for one phase of the work. Section I contains various

[1] *Popular Astronomy*, **25**, 666, 1917.

PLATE IV



THE GLOBULAR CLUSTER MESSIER 3 (N.G.C. 5272)

The side of a large square is 5,000,000 times the distance of the earth from the sun. The radius of the concentric circle corresponds to a parallax of 0".1 (2,062,650 astronomical units). To cross the circle light must travel for sixty-five years. Small circles contain typical variables; the small square, variable No. 37. If the sun were situated at the center, the Hyades would be at the distance of the triplet at the bottom of the picture; Sirius would be at the distance of the black cross near the center. A star of the luminosity of Sirius is indicated by the arrow-point. Stars of our sun's brightness are nearly two magnitudes too faint to appear on the photograph.

items relative to the method of investigation, comparative accuracy, maximum luminosities, and the frequency of giants.

2. The study of the distribution of clusters in space brings out a number of remarkable features (cf. sec. II), the most significant of which appear to be the absence from the denser stellar regions of globular clusters and the final proof that they are subordinate to the general galactic system. The center of the system of globular clusters is found, with some uncertainty in one co-ordinate. A discussion of the part played in a general theory by the distribution of clusters is postponed to a following paper.

3. The derivation of parallaxes has permitted the discussion of the actual dimensions of clusters and a comparison with familiar distances near the sun. Plate IV sufficiently summarizes the result for a typical system, Messier 3, whose distance from the earth is of the order of 250,000 million million miles. The total mass of a typical globular cluster is estimated to be from a quarter to a half of a million times the solar mass, with much uncertainty as to the upper limit.

MOUNT WILSON SOLAR OBSERVATORY
December 1917

ALPHA CENTAURI AS A SPECTROSCOPIC BINARY

By JOSEPH LUNT

Alpha Centauri has received a very large amount of attention from double-star observers since Lacaille first measured the position, angle, and distance of the companion in 1752. The stars have made two complete revolutions since Lacaille's first measures, and their orbits have been investigated by many astronomers. In 1893 T. J. J. See,[1] then of the University of Chicago, and Dr. A. W. Roberts,[2] of Lovedale, South Africa, published the elements of the orbit independently. In the words of the latter "the results are almost identical . . . . but as the two sets of elements are independent of each other and were obtained by very different methods, the coincidence can only indicate that the elements found are very near the truth."

In Fig. 1 the orbits of the two stars are shown according to Roberts' elements, the ascending and descending nodes being inserted in their proper positions derived from later spectroscopic observations.

The continuous-line ellipse and the right-hand scale, as well as the concentric circles and arcs, refer to the orbit of the fainter star (a_1) with respect to the brighter star (a_2) regarded as fixed in the center of the concentric circles.

The left-hand scale and the two ellipses taken together refer to the two stars as moving around their common center of mass situated at the center of the concentric circles, the longitudes from periastron at any instant being the same for both stars.

V_0, at longitudes 57° and 199°, indicates the positions in the orbits where the orbital radial velocity becomes zero, and the ascending nodes are the positions in the orbits where the *receding* radial motion becomes a maximum. The right-hand side of the diagram is therefore the near side, and the left the far side, of the

[1] *Monthly Notices*, **54**, 115, 1893.

[2] *Astronomische Nachrichten*, **133**, 106, 1893.

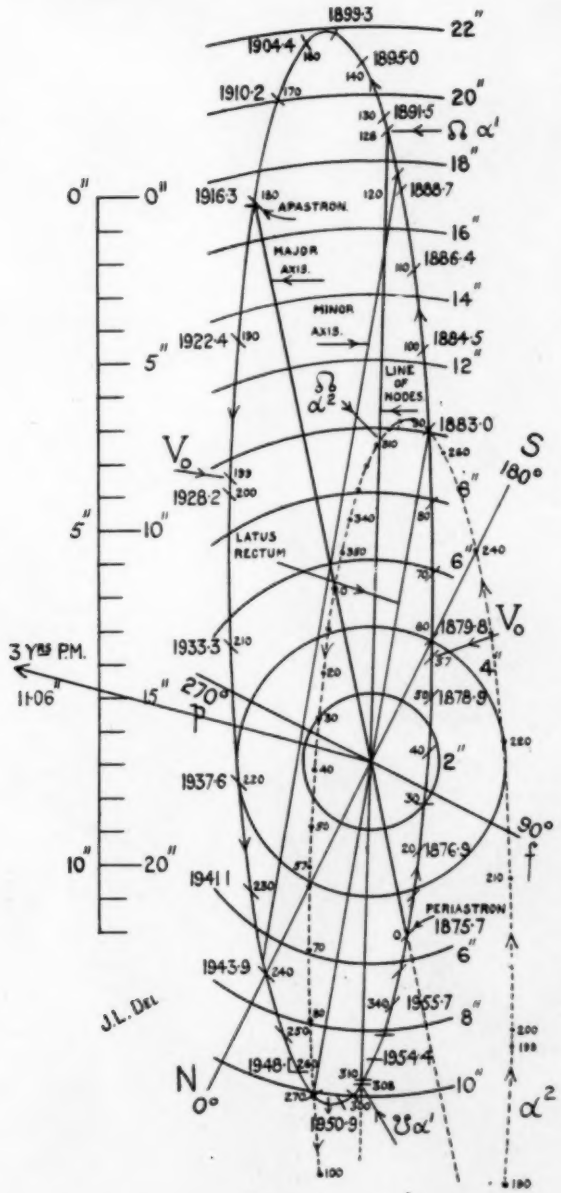

1899·3
22″
1904·4
1895·0
1910·2
20″
1891·5
Ω α′
18″
0″
0″
1916·3
APASTRON.
1888·7
16″
MAJOR AXIS.
1886·4
14″
MINOR AXIS.
1922·4
1884·5
5″
12″
LINE OF NODES
Ω α²
1883·0
V₀
S
180°
1928·2
8″
5″
10″
LATUS RECTUM
6″
3 YRS P.M.
1933·3
1879·8
V₀
11·06″
15″
270°
P
4″
1878·9
1937·6
2″
90°
f
10″
20″
1941·1
1876·9
PERIASTRON
1875·7
6″
1943·9
1955·7
8″
J.L. DEL
1948·0
1954·4
N
0°
10″
1950·9
Ω α′
α²

FIG. 1

orbits in space. The small figures represent longitudes from periastron in the orbits.

The figure has been drawn on the assumption of equal masses for the two stars. Roberts[1] gives the masses as $a_2 : a_1 = 51 : 49 \pm 2$ per cent. As the radial velocities are not appreciably affected at present by adopting 51:49 instead of equal masses, the latter value has been used throughout, as the greatest difference will be only 0.1 km in the positions of the maxima and minima of the velocity-curves.

Assuming a_2 and a_1 to be of equal mass, the semiamplitude of the velocity-curve[2] for both stars is

$$\frac{149500000\, a\, \pi \sin i}{365.25 \times 86400\, \pi'' P \sqrt{1-e^2}} = K$$

in nomenclature of spectroscopic binaries. The required data are

	Roberts	See	Doberck*
a	17.″71	17.″705	18.″165
i	79°21′36″	79°44′24″	79°19′
P	81.ʸ185	81.ʸ07	83.ʸ565
e	0.52865	0.52	0.52252

* *Astronomische Nachrichten*, **139**, 273, 1896.

Adopting Gill and Elkin's[3] value of the parallax, 0.″75, the values of K are

Roberts' elements	5.012 km/sec.
See's "	4.993 "
Doberck's "	4.971 "

The following elements based on Roberts' orbit have therefore been adopted

$$P = 29{,}652 \text{ days} = 81.^{y}18$$
$$T = \text{J.D. } 2406150 = 1875.71$$
$$\omega \begin{cases} a_2 = 52^\circ \\ a_1 = 232^\circ \end{cases}$$
$$e = 0.52865$$
$$K = 5.00 \text{ km/sec.}$$

[1] *Astronomische Nachrichten*, **139**, 10, 1896.

[2] See Campbell, *Astrophysical Journal*, **21**, 176, 1905.

[3] *Memoirs of the Royal Astronomical Society*, **48**, 188, 1884.

The orbital radial velocities of the stars will be:

α_2: $V_2 = +1.63$ km/sec. $+\frac{1}{2}^0$ cos $(v+52^\circ)$
α_1: $V_1 = -1.63$ km/sec. $+\frac{1}{2}^0$ cos $(v+232^\circ)$

where v is the true anomaly and $\pm 1.63 = Ke \cos \omega$. The orbital velocities at intervals throughout the paths have been computed and are given in the fourth column of Table I. Eliminating the orbital velocities from all the available observed radial velocities

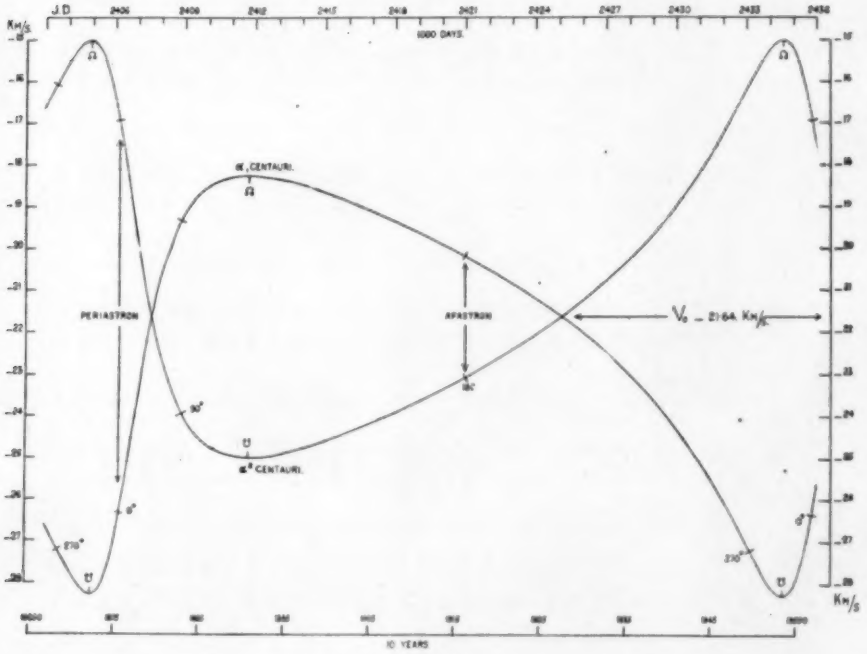


FIG. 2

given in Table II results in the weighted mean of the 28 values for the velocity of the system,

$$V_0 = -21.64 \pm 0.098 \text{ km/sec.}$$

Combining this value with the orbital motion gives the velocity ephemeris for the two stars found in the fifth and sixth columns of Table I and the velocity-curves of Fig. 2, where

$$\text{J.D.} = 2406150 + \tfrac{2}{3}\tfrac{0}{6}\tfrac{6}{0}\tfrac{5}{0}\tfrac{2}{2}m$$
(m = mean anomaly).

TABLE I

α Centauri (Ephemeris)

True Anomaly	J.D. 2400000+	Date A.D.	Orbital Rad. Vel. a_2 a_1	Radial Velocity		Difference of Rad. Vel.	Remarks
				a_2	a_1		
			km	km	km	km	
0°	6150	1875.71	±4.71	−16.93	−26.35	9.42	Periastron a_2 and a_1
10	6366	76.30	3.98	17.66	25.62	7.96	
20	6589	76.91	3.18	18.46	24.82	6.36	
30	6817	77.54	2.33	19.31	23.97	4.66	
40	7060	1878.20	1.46	20.18	23.10	2.92	
50	7330	78.94	±0.59	21.05	22.23	1.18	
57	7536	79.51	0.00	21.64	21.64	**0.00**	Zero orbital radial velocity
60	7630	79.76	∓0.24	21.88	21.40	0.48	
70	7965	1880.68	1.02	22.66	20.62	2.04*	
80	8356	81.75	1.72	23.36	19.92	3.44	
90	8819	83.00	2.31	23.95	19.33	4.62	
100	9373	84.53	2.78	24.42	18.86	5.56	
110	10049	1886.39	3.13	24.77	18.51	6.26	
120	10886	88.68	3.32	24.96	18.32	6.64	
128	11698	90.90	3.37	25.01	18.27	**6.74***	☊ a_1 ☋ a_2
130	11926	91.53	3.37	25.01	18.27	6.74	
140	13210	1895.04	3.26	24.90	18.38	6.52	
150	14779	99.34	3.01	24.65	18.63	6.02	
158	16244	1903.35	2.70	24.34	18.94	5.40	
160	16635	04.42	2.61	24.25	19.03	5.22	
162	17038	05.52	2.52	24.16	19.12	5.04	
164	17450	06.65	2.42	24.06	19.22	4.84	
166	17871	1907.80	2.31	23.95	19.33	4.62	
168	18298	08.97	2.20	23.84	19.44	4.40	
170	18734	10.17	2.09	23.73	19.55	4.18	
172	19173	11.37	1.97	23.61	19.67	3.94	

174	19621	1912.59	1.84	23.48	19.80	3.68	
176	20072	13.83	1.72	23.36	19.92	3.44	
178	20522	15.06	1.58	23.22	20.06	3.16	
180	20976	16.30	1.45	23.09	20.19	2.90	Apastron α_2 and α_1
182	21430	1917.55	1.31	22.95	20.33	2.62	
184	21880	18.78	1.17	22.81	20.47	2.34	
186	22331	20.02	1.02	22.66	20.62	2.04	
188	22779	21.24	0.87	22.51	20.77	1.74	
190	23218	1922.44	0.72	22.36	20.92	1.44	
192	23654	23.64	0.56	22.20	21.08	1.12	
194	24081	24.80	0.40	22.04	21.24	0.80	
196	24502	25.96	0.24	21.88	21.40	0.48	
198	24914	1927.09	∓0.08	21.72	21.56	0.16	
199	25116	27.64	0.00	21.64	21.64	**0.00**	Zero orbital radial velocity
200	25317	28.19	±0.08	21.56	21.72	0.16	
210	27173	33.27	0.93	20.71	22.57	1.86	
220	28742	1937.57	1.80	19.84	23.44	3.60	
230	30026	41.08	2.67	18.97	24.31	5.34	
240	31067	43.94	3.50	18.14	25.14	7.00	
250	31903	46.22	4.28	17.36	25.92	8.56†	
260	32579	1948.07	4.98	16.66	26.62	9.96	
270	33133	49.59	5.57	16.07	27.21	11.14	
280	33596	50.86	6.04	15.60	27.68	12.08	
290	33987	51.93	6.39	15.25	28.03	12.78	
300	34322	1952.84	6.58	15.06	28.22	13.16	
308	34565	53.51	6.63	15.01	28.27	**13.26**†	♌ α_1, ♌ α_2
310	34622	53.67	6.63	15.01	28.27	13.26	
320	34892	54.41	6.52	15.12	28.16	13.04	
330	35135	1955.07	6.27	15.37	27.91	12.54	
340	35363	55.70	5.87	15.77	27.51	11.74	
350	35586	56.30	5.35	16.29	26.99	10.70	
360	35802	1956.90	±4.71	−16.93	−26.35	9.42	Periastron α_2 and α_1

* First maximum difference of orbital radial velocity.

† Second maximum difference of orbital radial velocity.

TABLE II

Year	J.D.	No. of Plates	Measures		Standard Plate Used	Observed Rad. Vel.	Orbital Rad. Vel.	V_0 Reduced	Rad. Vel. Computed	Difference of Rad. Vel. O−C
			No.	By						
	α_1 Centauri (brighter star)									
						km	km	km	km	km
1904.....	2416577*	3	3	W	.....	−24.27	−2.62	−21.65	−24.26	−0.01
1904.....	16662	4	4	L	T	24.43	2.60	21.83	24.24	−0.19
			4	S	971	24.43	2.60	21.83	24.24	−0.19
			4	G	Z	23.95	2.60	21.35	24.24	+0.29
1906.....	2417345	7	7	S	971	22.55	2.44	20.11	24.08	+1.53
1906.....	411	4	4	S	971	23.01	2.43	20.58	24.07	+1.06
1906.....	443	14	13	H	971	24.17‡	2.42	21.75	24.06	−0.11
			13	H	T	23.70	2.42	21.28	24.06	+0.36
1907.....	626†	17	17	H	971	24.18‡	2.37	21.81	24.01	−0.17
			16	H	T	23.90	2.37	21.53	24.01	+0.11
1907.....	793	14	14	H	971	23.97‡	2.33	21.64	23.97	0.00
			14	H	T	23.00	2.33	20.67	23.97	+0.97
1907.....	794	1	1	S	971	22.95	2.33	20.62	23.97	+1.02
1908.....	2418013	10	10	H	971	23.90‡	2.27	21.63	23.91	+0.01
1908.....	052	7	7	S	971	24.99	2.26	22.73	23.90	−1.09
1908.....	163	11	11	S	971	24.83	2.24	22.59	23.88	−0.95
1908.....	171		5	H	2145	25.07	2.23	22.84	23.87	−1.20
1909.....	497	3	3	L	⊙	23.97	2.15	21.82	23.79	−0.18
1912.....	2419581§	1	1	L	⊙	24.24	1.86	22.38	23.50	−0.74
1912.....	635	4	4	H	2145	24.30	1.84	22.46	23.48	−0.82
1915.....	2420578	3	3	H	2145	−24.95	−1.56	−23.39	−23.20	−1.75
V_0 weighted mean.....								−21.66		

	α_1 Centauri (fainter star)									
1904.....	2416578*	3	3	W	.....	−19.10	+2.62	−21.72	−19.02	−0.08
1904.....	682†		3	L	T	17.97	2.60	20.57	19.04	+1.07
1904.....	682†	3	3	G	Z	17.47	2.60	20.07	19.04	+1.57
1909.....	2418497§	3	3	L	⊙	18.33	2.15	20.48	19.49	+1.16
1911.....	2419172§	3	3	L	⊙	20.63	1.97	22.60	19.67	−0.96
1912.....	632§	3	3	H	2145	21.32	1.84	23.16	19.80	−1.52
1915.....	2420585§	2	2	H	2145	−20.79	+1.56	−22.35	−20.08	−0.71
V_0 weighted mean	.....	.....	.....	.....	.....	.....	.....	−21.52	.....	.....

α_2 and α_1, V_0 weighted mean adopted, -21.64 ± 0.098

W=Wright; L=Lunt; S=Simpson; G=Goatcher; H=Halm; T=Toepfer measuring machine; Z=Zeiss comparator; standard plates: 971, α_2 Centauri; 2145, α Tauri. ⊙=Daylight.

* *Astrophysical Journal*, **20**, 141, 1904.

† *Annals of the Royal Observatory, Cape of Good Hope*, **10**, Parts I and III.

‡ A correction of −0.882 km/sec. has been applied to the published measures after remeasurement of the shift of the standard plate:
First standardization, 5 plates, JH, −2.012, *Cape Annals*, **10**, 40c.
Second standardization, 20 plates, JAS, -2.894 ± 0.093, *Cape Annals*, **10**, 50c.

§ Table III gives details of these measures.

REAL MOTION IN SPACE

Roberts[1] gives the following values for the annual proper motion:

$$\begin{aligned} &\text{In R.A.} && -7''.291 \pm 0.032 \ (1880) \\ &\text{In Dec.} && +0.750 \pm 0.005 \ (1880) \end{aligned}$$

The proper motion is therefore $3''.685$ toward $281°45'$, the component in R.A. being $-3''.608$ and in Dec. $+0''.75$. The correction for the solar motion to proper motions in R.A., expressed in seconds of arc on a great circle, $= -3''.656\,\pi \cos \alpha$, and in Dec. $= +3''.656\,\pi \sin \alpha \sin \delta + 2''.111\,\pi \cos \delta$, where π is the star's parallax and $+3''.656$ and $-2''.111$ are the apparent proper motions, in R.A. and Dec. respectively, of a star with a parallax of $1''$ at 0^h in the equator, due to the solar motion assumed to be 20 km/sec. toward 18^h and $+30°$.

The corrected proper motions are therefore,

In R.A.		In Dec.	
	$-3''.608$		$+0''.750$
	$+2.165$		$+2.245$
	-1.443		$+2.995$

which combined give a corrected proper motion of $3''.325$ toward $334°16'$ (1880),

$$\begin{aligned} 3.685 \cdot \frac{4.737}{\pi} &= 23.27 \text{ km/sec.} = \text{apparent proper motion} \\ 3.325 \cdot \frac{4.737}{\pi} &= 21.00 \quad \text{“} \quad = \text{corrected proper motion} \\ \hline 2.27 \quad &\text{“} \quad = \text{solar component} \end{aligned}$$

where 4.737 = the velocity across the line of sight, in kilometers per second, of a star with $\pi = 1''.0$ and proper motion $= 1''.0$. The correction to radial velocities for the solar motion

$$\begin{aligned} &= \eta \frac{\Delta Y}{3460.185} + \zeta \frac{\Delta Z}{3460.185} \\ &= 0.0050055\eta - 0.002895\zeta. \end{aligned}$$

[1] *Astronomische Nachrichten*, **139**, 10, 1896.

where ΔY and ΔZ are the reversed components of the solar motion, i.e., toward 6^h and $-30°$ respectively, expressed in kilometers per second; $\eta = -c \sin \alpha \cos \delta$ and $\zeta = -e \sin \delta$; c and 3460.185 = the factor for converting astronomical units per twelve hours into kilometers per second.

Adopting the value -21.64 km/sec. for the radial velocity of the system with respect to the sun, and the solar-motion correction -3.41 km/sec., the real motions are

Radial	-25.05	Combined 32.7 km/sec.
Transverse	21.00	

The system of α Centauri is therefore approaching the sun in a direction inclined 40° to the line joining the sun and star, with a velocity of 32.7 km/sec., regarding the sun as stationary. Combining the sun's motion, the velocity becomes 31.8 km, and the angle 47°.

Alpha Centauri has been under observation spectroscopically only since 1904, and only a small part of the velocity-curve has been determined by observation. In Table II are collected all the observations available. It is self-explanatory.

Table III gives the individual measures not previously published, compared with the computed velocities.

The velocity-curve from See's elements in the region of the spectroscopic observations is practically identical with that from Roberts' elements, and that from Doberck's elements differs only from one- to two-tenths of a kilometer for the same region, Doberck's negative velocities being slightly larger for α_2.

The observations for α_2 Centauri given in Table II may be arranged in the groups indicated by brackets, in which the individual results are accordant.

The accordance within the groups is quite satisfactory, but the groups differ from each other somewhat markedly; they lie within a zone 3.28 km wide, with the velocity-curve almost central. Whether the divergencies from the theoretical velocities are due to instrumental causes or represent real changes has not been determined. The later measures show a distinct tendency to be too low, the mean difference O−C for the plates given in Table III

TABLE III

Plate No.	Date	Sidl. Time	Correction to Sun	Rad. Vel. Observed	Rad. Vel. Computed	Diff. O−C	Measured by
			α_2 Centauri				
			km	km	km	km	
1770	1908 July 24	$17^h 30^m$	−19.45	−24.79	−23.88	−0.91	JAS*
1773	25	15 20	19.50	23.78	23.88	+0.10	JAS
1774	25	15 39	19.52	24.12	23.88	−0.24	JAS
1775	25	16 3	19.54	25.44	23.88	−1.56	JAS
1811	Aug. 15	17 8	21.47	23.49	23.88	+0.39	JAS
				25.03	23.88	−1.15	JH
1812	11	17 48	21.50	25.71	23.88	−1.83	JAS
1815	12	17 0	21.53	25.69	23.88	−1.81	JAS
				25.27	23.88	−1.39	JH
1821	13	17 19	21.59	25.49	23.88	−1.61	JAS
				25.75	23.88	−1.87	JH
1845	21	17 25	21.81	24.56	23.87	−0.69	JAS
				24.56	23.87	−0.69	JH
1861	27	17 3	21.69	24.51	23.87	−0.64	JAS
				24.74	23.87	−0.87	JH
1862	27	17 19	21.70	25.54	23.87	−1.67	JAS
2398	1909 June 25	12 35	12.28	24.10	23.79	−0.31	JL
2417	July 16	16 2	17.84	23.32	23.79	+0.47	JL
2422	17	14 38	17.96	24.48	23.79	−0.69	JL
3674	1912 June 27	13 44	12.98	24.24	23.50	−0.74	JL
3759	Aug. 10	16 9	21.38	24.15	23.48	−0.67	JH
3760	10	16 28	21.39	23.49	23.48	−0.01	JH
3779	30	17 7	21.54	24.60	23.48	−1.12	JH
3780	30	17 36	21.55	24.96	23.48	−1.48	JH
4515	1915 Mar. 8	11 30	+21.14	24.26	23.21	−1.05	JH
4528	25	12 59	+18.10	24.97	23.20	−1.77	JH
4539	31	13 21	+16.65	−25.61	−23.20	−2.41	JH
Mean diff. O−C						−0.97	

	α_1 Centauri						
2399	1909 June 25	13 21	−12.32	−18.23	−19.49	+1.26	JL
2418	July 16	16 39	−17.86	18.41	19.49	+1.08	JL
2423	17	15 14	−17.99	18.35	19.49	+1.14	JL
3123	1911 May 5	14 56	+ 5.53	20.62	19.67	−0.95	JL
3140	15	13 46	+ 1.99	21.93	19.67	−2.26	JL
3151	26	13 34	− 2.02	19.35	19.67	+0.32	JL
3761	1912 Aug. 10	17 9	−21.40	20.46	19.80	−0.66	JH
3762	10	18 2	−21.44	21.56	19.80	−1.76	JH
3781	30	18 14	−21.57	21.94	19.80	−2.14	JH
4529	1915 Mar. 25	13 31	+18.04	20.09	20.08	−0.01	JH
4538	31	12 38	+16.68	−21.48	−20.08	−1.40	JH
Mean diff. O−C						−0.41	

JAS, standard plate, α_2 Centauri 971.
JL, solar (daylight) standards.
JH, standard plate, α Tauri 2145.

* Second Lieutenant J. A. Simpson, R.F.C., killed in action, France, October 23, 1916.

being -0.97 and -0.41 km per second for a_2 and a_1 respectively. The velocity of the system adopted, viz., -21.64 km, gives velocities agreeing with Wright's 1904 observations almost exactly but is half a kilometer higher than his later value -22.18 km.[1]

It will be seen from the figures that the distance between the two stars is slowly decreasing until 1937.6, when they will be 4″ apart; the distance will then increase until 1951, when they will be over 10″ apart, after which time the distance will diminish to less than 2″ in 1959.

The difference of radial velocity is gradually diminishing and will be zero in 1927, when spectroscopic observations will be very desirable, as both stars will have the radial velocity of the system. After that date the difference of radial velocity will rapidly increase until 1953, when the difference will be over 13 km/sec.

ROYAL OBSERVATORY, CAPE OF GOOD HOPE
May 16, 1918

[1] *Lick Observatory Bulletin*, **6**, 27.

THE NATURE OF A SUPPOSED VARIATION IN THE SOLAR ROTATION IN 1915

By RALPH E. DeLURY

In a brief note[1] evidence was presented which indicated that the conclusion[2] drawn by Mr. H. H. Plaskett "that the sun, during the summer of 1915, underwent a cyclic variation in its rotation rate of 0.15 km . . . . completed in about a month," was not justified, and that this variation in spectroscopic measurements of the velocity of rotation of the sun's equatorial limbs might be due to changes in terrestrial haze. Mr. Plaskett has criticized[3] this explanation without testing my suggestion that remeasurements or an analysis of old measurements should show (on account of the presence of blended spectrum of haze in the observations in question) smaller displacements of weak than of strong spectrum lines, particularly on the days when the lowest values of the rate of rotation were obtained, i.e., on the days of greatest haziness. In what follows new evidence confirming this explanation is given along with an elucidation of the criticisms, which are referred to by their numbers.

1. From a considerable number of observations with respect to haze Mr. Plaskett selected three of my observations, implying that these were all I had made. He places these observations of obviously changed conditions beside his measurements of plates made some hours previously, implying in the words "low" and "high" that the haze explanation has been contradicted, thus:

> During the period under consideration, namely, June 21 to August 16, DeLury made three entries in the observation book with reference to haze:
>
> June 21, "Bright" ..... $V=1.911$ (low)
> July 11, "Very hazy" ..... $V=1.975$ (high)
> August 16, "Bright, some water-vapor haze" ..... $V=1.977$ (high)

[1] *Astrophysical Journal*, **44**, 198, 1916.

[2] *Ibid.*, **43**, 156, 1916.

[3] *Ibid.*, **45**, 144, 1917.

Instead of this he should have adopted the logical procedure of juxtaposing as follows his own observations with reference to haze and the respective values of the velocity of rotation, finding in accordance with the haze explanation that low values of the measurements of solar rotation are associated with haze and higher values with "no haze," thus:

	V in km per sec.
June 24, "B, 3–4, A little haze near sun"	1.846
July 4, "B, 3–4, Haze"	1.893
July 20, "B, 4, No haze"	2.026
July 23, "B, 4–5, Positively no haze"	2.003
August 16, "B, 4, Practically little or no haze"	1.977

That he committed an error in setting his measures of rotation over against my records of changed conditions made later in the day is apparent from a consideration of Table I, where his and my measurements for these three selected days are associated with their respective records of brightness, the observations being in harmony with the haze explanation.

TABLE I

Date	Plaskett			DeLury			
	Time	Brightness	Km per sec.	Time	Brightness	λ	Km per sec.
June 21....	6:45– 8:00	4	1.911	10:18–10:38	Bright	6450	2.007
July 11....	7:00– 7:19	4, "then hazed up"	1.975	9:26–10:58	Very hazy, cirrus clouds	5900	1.785
				11:37–11:49	Brighter, still hazy	5600	1.805
August 16..	8:40–11:49	4, "practically little or no haze"	1.977	12:47–12:54	Bright, some water-vapor haze	5600	1.939

It is significant that on June 21, when he obtained a value of 1.911 km per sec.—one of the low values in the "cyclic variations"—I obtained a few hours afterward 2.007 km per sec., a value about equal to the maximum values in the supposed variation. Probably the record "Bright" indicated a greater brightness than "B, 4" of that date.

On account of the haze which developed after Plaskett's plates were taken on July 11 and which remained throughout my observations these latter (14 plates at λ 5900 and 7 at λ 5600) yield decidedly lower values than his.

On August 16, probably on account of the presence of "some water-vapor haze," my observations (7 plates at λ 5600) yield a value 2 per cent lower than his.

All these results are ascribable to different degrees of haziness in accordance with the records. Further details are given in Table II.

Regarding his record of brightness Mr. Plaskett states:

> 5 represents a very brilliant day—rare in Ottawa.
> 4 represents a bright day—normal observing weather.
> 3 represents a day slightly hazy.

These statements are self-contradictory, presuming that such different degrees of brightness postulate different degrees of haziness. On no day was haze absent. At that time our records of observing conditions were incomplete in that no account was kept of the strength of the spectrum of haze. During the past two years on no day has it been impossible to see the lines in the spectrum from haze just outside the solar limb (at λ 5600, third order, 7 m auto-collimating [5 cm×8 cm] grating spectrograph, slit 0.05 mm, fed by the 45 cm concave mirror, 24.5 m focus). Usually the lines are distinctly visible. Again he states, "Plates taken on days hazier than 3.5 were *not* measured." This is a misreading of the record, as 14 of the 111 plates measured by Plaskett were taken at recorded brightness "3."

Plaskett also asserts: "The record, if it shows anything, shows that in general high values were obtained on hazy days and low values on brighter days, in direct contradiction to DeLury's statement."

That this conclusion is fallacious is seen from the following complete summary of his observations (though it is something to have the admission that there were hazy days). The measurements of the 111 plates may be grouped with regard to brightness "above normal," "normal," and "below normal," using Plaskett's definition of a normal day given above:

	km per sec.
19 plates, B, "5" and "4–5," mean velocity	1.975
50 plates, B, "4," i.e., "normal," mean velocity	1.958
42 plates, B, "3–4" and "3," mean velocity	1.942

It may be noted that 42 of the 111 plates were made on days of brightness below "normal," i.e., on days of haze presumably above normal. Also 43 of the 111 plates were taken at brightnesses recorded as "4–5" and "3–4," terms so vague as to indicate either uncertainty in the estimates of brightness or varying degrees of haziness. When such uncertainty in the records exists, the support which the summary gives to the haze explanation is as satisfactory as can be expected.

The 19 plates of the first group include 15 at "B, 4–5," mean velocity 1.989 km per sec., and 4 at "B, 5," mean velocity 1.919 km per sec. The accuracy of this high record of brightness (the only one of "B, 5") may be questioned, as it was the first made (on July 9) by Plaskett after the three mirrors of the coelostat telescope had been freshly plated (July 7), and it is probable that the much brighter image of the sun produced as a consequence gave the observer an exaggerated impression of the clearness of the sky. If the day were in reality hazy, then the plates should satisfy the criterion for blended spectrum of haze. I have measured the 4 plates and find a difference of 0.054 km per sec. between the values of the velocity of rotation deduced from groups of weak and of strong spectrum lines—a difference which may be interpreted as indicating the presence of a considerable degree of haziness (see Table II). Hence it is reasonable to suppose that record "B, 5" for the brightness during his observations on July 9 is too high relative to the other records and for the cause indicated.

From the foregoing summary of his records and measurements it is quite evident that the following statement which appears in his original paper (p. 147) is not justified: "Observations were never made except on days that were free from haze."

2. Mr. Plaskett states in his note: "DeLury bases his discussion on plates made on two dates, June 24 and July 20, which are described as 'Hazy' and 'Bright,' apparently from memory, as it is not so recorded in the observation book."

On June 24 he recorded "B, 3–4, a little haze near sun," at 7:40 A.M. before his two observations, and "B, 3–4" at 9:10 A.M. after these observations. Memory need not be appealed to for the description "Hazy" for these conditions. On July 20 he recorded

"Bright (4)" three times for intervals 6:23–7:08, 8:03–8:24, and 8:38–8:43 A.M.; also, "No haze." I observed from 9:54–10:11 A.M., allowing these records to stand until a change occurred just after this, when I recorded that it "clouded up." Hence the term "Bright" is justified, "4" representing, according to him, a "bright day, normal observing weather."

In the following two misstatements are made: "On June 24 DeLury's plates were taken through clouds whose presence he notes in the observation book, whereas the writer's plates, exposed earlier, were taken with a clear sky." On this date my measured plates at λ 4230 were taken at 10:18–10:54 A.M., and though I made observations up to noon I did not record "clouded up" until afternoon. Absolutely no mention of clouds was made before noon, so that the statement that my "plates were taken through clouds" is not correct. Also, as shown above, on June 24 he described the conditions as "a little haze near the sun" and "B, 3–4," so that the term "clear sky" by his own definition is incorrect.

In addition to the direct statement of the records as to the relative brightness of the two selected dates, June 24 and July 20, remeasurements of my plates at λ 4230 and measurements by myself of his plates at λ 5900 yield further confirmation of my explanation, for on June 24 the differences between groups of measurements of weak and strong lines are greater than for the same lines on July 20, pointing to a greater degree of haziness on the former date (see Table II).

TABLE II

Date	Time	Brightness	No. of Plates	λ	No. of Lines	Mean Intensity	Velocity in km per sec	No. of Lines	Mean Intensity	Velocity in km per sec	Difference in Velocity
June 24	8:17–9:10	3–4	2	5900	11	3.7	1.760	3	11	1.855	0.095
	10:18–10:54	.....	7	4230	2	2.5	1.714	1	20	1.857	0.143
July 9	8:03–8:22	5 (mirrors replated)	4	5900	11	3.7	1.888	3	11	1.942	0.054
July 11	10:25–10:58	Very hazy	14	5900	11	3.7	1.767	3	11	1.852	0.085
	11:37–11:49	Brighter, still hazy	7	5600	3	0.3	1.783	3	5.3	1.826	0.043
July 20	6:23–8:03	4, no haze	4	5900	11	3.7	1.994	3	11	2.022	0.028
	9:54–10:11	.....	5	4230	2	2.5	1.982	1	20	2.004	0.022
Aug. 16	12:47–12:54	Bright, water-vapor haze	7	5600	3	0.3	1.939	3	5.3	1.939	0.000

Table II gives in detail the new measurements discussed above. They are all in harmony with the haze explanation. (My 14 plates

at λ 5900, July 11, yielded a velocity of −0.006 km per sec. for the means of 4 atmospheric lines, showing that little error need be feared when the proper precautions in focusing the plate and in illuminating the grating evenly are taken.)

Regarding the possibility of errors of measurement in the results given in my note Mr. Plaskett says: "The difficulties of measurement of broad lines increase the errors and chances of prepossessions, especially with an observer whose last published measures show a probable error for even well-defined lines of 0.06 km, i.e., of the same order as the differences measured."

There are no "chances of prepossessions," for in all of my spectroscopic measurements there is used a movable mask,[1] which, by permitting only one strip of spectrum to be seen at a time, keeps hidden not only the magnitude but also the direction of displacement of the spectrum lines being measured.

In the series referred to the average probable error was 0.052 km per sec. (not 0.06), and in this very series numerous repeated measures of a certain plate have previously shown that about half of this "probable error of a single line" is due to actual differences for different lines on the plate. Also the plates used in the series referred to are of coarser grain and the definition of the lines is poorer than for the plates at λ 4230, referred to in my note.

In Table II are given the means of six measurements of the latter plates (λ 4230, June 24 and July 20), including the single measures reported in my note—three violet right and three violet left. The former single measure differs from these means by 0.002 and 0.013 km per sec. for the Fe lines, and 0.063 and 0.021 km per sec. for the broad Ca line; or, expressed in millimeters, these quantities are 0.0001, 0.0005, 0.0024, and 0.0008. The differences in velocity between the Ca and the Fe lines, namely, 0.143 and 0.022 km per sec., point to an even greater effect of blended spectrum of haze than was supposed from the differences 0.082 and 0.014 km per sec. given in the previous note.

It bears strictly on the whole question to compare these differences, based on measures of a single broad line, with Mr Plaskett's

[1] *Journal of the Royal Astronomical Society of Canada*, 5, 405, 1911.

two-way measurements of *five* solar lines on 111 plates, which when repeated yielded an average difference between the means of 0.055 km per sec., of which differences 18 exceeded 0.100 km per sec. and 6 have a mean of 0.152 km per sec. of the same magnitude as the "cyclic variation of 0.15 km per sec."

The differences derived from the means of the new measurements of the λ 4230 plates of June 24 and July 20 effectively annul the following argument (even if we overlook the fact that it is fallacious to argue as he has done from my measurements[1] at λ 5200 to those of other lines at λ 4230 without previously comparing the extent of the change in character of the two sets of lines in passing from the center of the solar disk to the limbs):

> The measures show evidence, according to the criterion of varying velocity for different line intensity, of about 8 per cent haze. This, on his own hypothesis, should produce about an 8 per cent difference in velocity on the two dates. The difference in mean velocity is actually 12 per cent, leaving a 4 per cent change in velocity to be accounted for.

3. The arguments as to the differences in value of the rotation for low sun and slightly higher sun are based on "meager and inconclusive" data, and the quantities derived are so very small (a few thousandths of a km per sec.) that they are without meaning when such large errors of measurement are present. A very large series of observations of the solar rotation from early morning to late afternoon would give evidence as to the daily development (usually increasing at Ottawa) of haze during that day, and if many days were likewise considered a valuable statement could be made as to the daily development of haze at the locality of the observations. No such measurements could possibly cast doubt on the haze explanation unless at the same time contradictory evidence of the changes in haze were obtained by other reliable means.

It may be said, in conclusion, that the facts presented above make the claim that the sun's rate of rotation varied in 1915 seem very unreasonable. All others who have made spectroscopic investigations of the solar rotation have found variety in their measures, and some have suggested the possibility that the rate

[1] *Astrophysical Journal*, **44**, 184, 1916.

varied, e.g., C. A. Young,[1] Halm,[2] Evershed and Royds[3], St. John, Adams, and Ware.[4] However, Halm's low values in 1902–1903 can be readily accounted for by the haziness produced by the volcanoes in the West Indies in 1902, which caused a marked lessening in the values of solar radiation. The low values obtained in 1912–1913 by Evershed and Royds may have been due to the Katmai eruption in June 1912, which also caused a great lessening in the measures of solar radiation. The low values of rotation obtained at Mount Wilson and Ottawa in 1915 also seem to synchronize with low values of radiation, i.e., it may be presumed, with high values of haziness. Apart from haze various instrumental and physical causes produce variations in the measurements of solar rotation. When the effects of all known causes (haze, local convections in the sun, errors of measurement, etc.)[5] are eliminated or accounted for, it probably will be possible to assert that the rate of the sun's rotation does not appreciably vary.

The foregoing may be summarized briefly as follows:

1. The arguments which Mr. H. H. Plaskett advances (in his note criticizing my explanation of his observations of a supposed "cyclic variation" in the sun's rate of rotation in 1915, as due to variations in terrestrial haze) are based on fallacious use, or inaccurate statement, of the records and the measurements.

2. A synopsis of Mr. Plaskett's record of observations and measurements of the solar rotation supports the conclusion that in general high values of the rotation were obtained during brighter conditions and low values during hazier conditions.

3. Measurements of Mr. Plaskett's plates, as well as of mine, satisfy the criterion of blended spectrum of haze in agreement with the record of observational conditions.

4. The *known* terrestrial sources of blended spectrum offer a reasonable explanation of the supposed variation in rotation, so

[1] *The Sun* (1882), p. 100.

[2] *Transactions of the Royal Society of Edinburgh*, **41**, 89, 1904; and *Astronomische Nachrichten*, **173**, 287, 1906.

[3] *Monthly Notices*, **13**, 554, 1913.

[4] *Popular Astronomy*, **23**, 641, 1915.

[5] DeLury, *Astrophysical Journal*, **44**, 178, 1916.

that it is unnecessary to employ the *hypothetical* element of my explanation, which supposes an interplanetary or solar source of blended spectrum.

5. At present there is no sound reason for believing that the rate of the sun's rotation is variable.

I wish here to thank the director, Dr. Klotz, for his approval of the publication of this note.

SOLAR PHYSICS DIVISION
DOMINION OBSERVATORY, OTTAWA
May 1918

ERRATA

Vol. **44,** November 1916, "On the Orbits of the Spectroscopic Binaries α Orionis and α Scorpii," by Joseph Lunt:

Page 251, ninth line, *for* what *read* that.

" 257, Table II, last column, third entry, 0.00 *align with* Mean, 2 plates.

Vol. **46,** September 1917, "Preliminary Examination of the Planetary Nebulae for Preferential Motion," by C. D. Perrine:

Page 176, tenth line of Table II, *for* B, vS *read* B, S

Vol. **46,** November 1917, "Relation to Proper Motion of Preferential Motion and of the Progressions of Spectral Class and Magnitude-Velocity," by C. D. Perrine:

Page 278, eighth line, *for* B. Boss *read* Lewis Boss.

Vol. **48,** July 1918, "The 'Astronomical Atom' and the Spectral Series of Hydrogen," by Fernando Sanford:

Page 6, equation (8),

$$\textit{for } R=\frac{3.503\cdot 10^{-6}}{\sqrt{\lambda}} \quad \textit{read } \frac{R}{\sqrt{\lambda}}=3.503\cdot 10^{-6}$$

Vol. **48,** September 1918, "The Absorption of Near Infra-Red Radiation by Water-Vapor," by W. W. Sleator:

Page 127, in legend, *for* scale 1 to 5 *read* scale 1 to 9.